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| **Unit Title:**Trigonometric Identities and Equations | **Grade Levels:** 11th grade |
| **Subject/Topic Area:** Using trigonometric identities and solving trigonometric equations |
| **Key Words:** Trigonometric Identities, Solving Trigonometric Equations, Right Triangles, Trigonometric Ratios, Area, Law of Sines, Law of Cosines, Angle Identities, Double-Angle Identities, Half-Angle Identities |
| **Designed by:** Jennifer Singleton | **Time Frame:** 12 days (classes are 45 minutes)  |
| **School District:** Specific County | **School:** Randomville High School |

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| **Brief Summary of Unit (including context and unit goals):** **This section should be completed only after all other sections of the unit have been written.** |
| For this unit, we are learning about trigonometric identities, trigonometric ratios, law of sines, law of cosines, and angle identities. The students will learn how to use different types of trigonometric identities when verifying expressions or solving equations. There will be worksheets, a quiz, group work, a performance task, and a unit test that will be given to access the students’ understanding on the material covered. Technology will be utilized to maximize student understanding and engagement. By the end of the unit, the students should be able to identify, verify, and use all of the trigonometric identities, trigonometric ratios, and other terms in the chapter.  |

**Identify Desired Results**

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| **Established Goals:** |
| * 36.) Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1, and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle. [F-TF8]
* 34.) (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. [F-TF6]
* 35.) (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.\* [F-TF7]
* 19.) Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle leading to definitions of trigonometric ratios for acute angles. [G-SRT6]
* 21.) Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.\* [G-SRT8]
* 22.) (+) Derive the formula *A* = (1/2)*ab* sin(*C*) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. [G-SRT9]
* 23.) (+) Prove the Law of Sines and the Law of Cosines and use them to solve problems. [G-SRT10]
* 24.) (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). [G-SRT11]
* 36.) (+) Prove the addition and subtraction formulas for sine, cosine, and tangent, and use them to solve problems. [F-TF9]
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| **What understandings are desired?** |
| * Students will use trigonometric identities can be used to simplify and verify other trigonometric expressions.
* Students will use trigonometric concepts can be used to solve triangles.
* Students will illustrate that with domain restriction, trigonometric functions can have equivalent values to trigonometric ratios for a right triangle.
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| **What essential question(s) will be considered?** |
| * How do you verify that an equation involving the variable x is an identity?
* A trigonometric function corresponds one number to many, so how can its inverse be a function?
* How do the trigonometric functions relate to the trigonometric ratios for a right triangle?
* How can trigonometric identities be used to find unknown sides and angles in triangles, including angles of depression and elevation?
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| **What key knowledge and skills will students acquire as a result of this unit?** |
| *Students will know …… (this should be stated with a noun)** Trigonometric Identities
* Right Triangle
* Trigonometric Ratios
* Area of Triangle
* Law of Sines
* Law of Cosines
* Angle Identities
* Double-Angle Identities
* Half-Angle Identities
 | *Students will be able to ……… (this should be stated with a verb that describes what students will be able to do.)** Verify trigonometric identities
* Evaluate inverse trigonometric functions
* Solve trigonometric equations
* Determine lengths of sides and measures of angles in a right triangle
* Determine the area of any triangle
* Verify and use the different types of angle identities
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**Determine Acceptable Evidence**

Relate all evidence to the goal(s), understanding(s), and/or essential question(s) being assessed.

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| **What evidence will show that students understand?** |
| *Performance Tasks: Also complete a Task Blueprint for each performance task listed.** Students will be split into groups of two. Each group will receive a packet with 4 tasks that must be completed. In the first task, the students will be given a picture of a mountain with different unmarked runs. The students will be given a range of degrees and what kind of steepness defines each range (runs). The students’ objective is to use their trigonometric skills to calculate the steepness of each run by finding its angle of elevation. In the second task, the students have to figure out the route that would be the easiest run to ski down. In the third task, students will be given a scenario in which owners of a ski resort want to build a row of condos that model their chalets. The students will be given a drawing of a chalet with certain lengths, angles, and stipulations. They will have to figure out the slant height of the roof on the chalet and what the slant height of the roof will be on one of the condos. In the fourth task, students will be given a picture of two different sized mountains, the altitude of one mountain, and two degree measures. They will have to figure out how much higher one mountain is than the other.
* I expect students to understand that the information they use in this activity can and is used in certain aspects of real life.
* See attached file for more instruction.
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| **Other Evidence (quizzes, tests, prompts, observations, dialogues, work samples):**  |
| ***Formative assessments*** * Warm-ups over information students should already know that will prepare them for the lesson.
* Observations made by the teacher to see what students do and do not understand.
* Questioning from teacher to students to probe them to come up with answers on their own.
* Discussion on why learned information works and how it relates to prior learned information.
* Class work over information learned through each lesson.
* Group work over prior learned information.
* Quiz over first three sections to access student understanding.
* Homework over information learned through each lesson.

***Summative assessments*** * Quiz over first three sections
* Homework from each lesson (three of the problems will be graded for accuracy)
* Performance Assessment
* End of unit test over information through each section.
 |
| **Student Self-Assessment and Reflection:** |
| ***List all activities throughout the unit when students will be expected to self-assess and self-reflect. These should be points when students are reflecting on what they are learning and on how they can learn more, not simply looking at what they have completed and reviewing their grades.**** Homework – Students will be given back homework’s with low scores. They will be put in groups of two or three, with students whom also scored low on the same homework, and will discuss mistakes and redo similar problems together.
* Quiz – Write a journal entry on how they did on their quiz and reflect on how they could have done better.
* Performance Task – Students will have to self-assess on what they have learned in order to complete tasks within the performance assessment.
* End of unit test – After each unit test, students will be asked to write two things they learned and two things they did not understand about the topic the test covered.
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| **Performance Task Blueprint –** ***You should complete this for each performance task.*** |
| **What understandings and goals will be assessed through this task?** |
| *Understanding(s):** Students should realize that applications of trigonometry are used in everyday life, and if/how they use trigonometry applications in their own life.
* Students should understand that identities can be used to verify and solve equations.
* Students should understand that trigonometric functions relate to trigonometric ratios for right triangles.
* Students should understand that trigonometric identities can be used to find unknown sides and angles of triangles
 | *Goals:* * Students will be able to apply their knowledge of trigonometry to a real life situation.
* Students will be able to apply their knowledge of trigonometric identities to verify equations, find unknown angles, and find unknown sides.
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| **What criteria are implied in the standards and understandings *regardless* of the task specifics?** **What qualities must student work demonstrate to signify that standards were met?** |
| *Criteria implied in the standards and understandings:* Students must know how to use certain trigonometric identities. Students must also know how to use certain trigonometric ratios. Students should also know how to set up proportions.  | *Qualities of student work: Should reflect your rubric for evaluation.*Students should show their work that leads them to their answers for each task. Even if answers are not correct, student work should indicate their knowledge of the material. Student work should show their pathways to answers.  |
| **Through what authentic performance task will students demonstrate understanding?** |
| * The students will complete a performance task on the 6th day of the unit. The students will be split into groups of two. Each group will be given a worksheet packet that involves four tasks. In the first task, the students will be given a picture of a mountain with different unmarked runs. The students will be given a range of degrees and what kind of steepness defines each range (runs). The students’ objective is to use their trigonometric skills to calculate the steepness of each run by finding its angle of elevation. In the second task, the students have to figure out the route that would be the easiest run to ski down. In the third task, students will be given a scenario in which owners of a ski resort want to build a row of condos that model their chalets. The students will be given a drawing of a chalet with certain lengths, angles, and stipulations. They will have to figure out the slant height of the roof on the chalet and what the slant height of the roof will be on one of the condos. In the fourth task, students will be given a picture of two different sized mountains, the altitude of one mountain, and two degree measures. They will have to figure out how much higher one mountain is than the other. The students should be able to complete these tasks in the 45 minute time frame, however, if they do not finish, the groups may evenly divide up the remaining problems and finish it for homework (If they cannot work on it together outside of school). I am using this performance task not only to access the students’ understanding of when they should use certain identities or ratios, but also to see the different methods students choose to use on these four tasks.
 |
| **What student products and performances will provide evidence of desired understandings?** |
| *Student products:* * Students will complete each of the four tasks and provide proof to how they solved each task.
* Calculate steepness and lengths of ski runs on a mountain. Calculate slant height of a chalet and the roofs of condos. Calculate which of two mountains is higher. (see attached assessment.
 | *Student performances:* * Students will have to use their trigonometric knowledge to find lengths and angles of certain objects
* Use trigonometric ratios to produce solutions to given questions
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| **By what criteria will student products and performances be evaluated?** *(The following should be completed in the form of a rubric(s) to evaluate students’ product(s) and/or performance(s).* |
| *Criteria for student product evaluation:*

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|  | **Exemplary** | **Adequate** | **Inadequate** |
| **Subject Knowledge/Content** | 40-45 pointsSubject knowledge is outstanding; shows in-depth understanding of the content with unique insights. | 20-39 pointsSubject knowledge is evident; shows appropriate understanding of the subject/content. | 0-19 pointsSubject knowledge is lacking; shows limited understanding of the content.  |
| **Organization/Grammar** | 12-15 pointsThoughts are well organized with few or no grammatical errors. Problem-solving is well structured  | 6-11 pointsThoughts are organized with some grammatical errors. Problem-solving is somewhat structured. | 0-5 pointsThoughts are not organized and there are many grammatical errors. Problem-solving has no structure. |
| **Thinking Levels** | 25-30 pointsDemonstrates concepts/standards through exceptional use of analysis and creative thinking that incorporates complex thinking.  | 15-24 pointsUtilized adequate application of concepts/standards involving analysis. | 0-14 pointsDoes not go beyond knowledge and comprehension levels.  |
| **Overall** | 10 pointsProduct goes beyond requirements. Demonstrates superior insights.  | 7-9 pointsProduct meets the requirements of the assignment.  | 0-6 pointsProduct does not meet requirements of the assignment.  |

 | *Criteria for performance evaluation:*

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|  | **4** | **3** | **2** | **1** |
| **Correct Solution** | All four tasks have the correct solution | Three of the tasks have the correct solution | Two of the tasks have the correct solution | One of the tasks has the correct solution |
| **Procedural Knowledge** | Selects and implements an appropriate strategy. Correctly implements procedure to arrive at four correct solutions. | Selects and implements an appropriate strategy. Implements selected procedure but arrives at one incorrect solution. | Selects and implements an appropriate strategy. Implements selected procedure but arrives at two incorrect solutions. | Selects and implements an inappropriate strategy. Makes significant errors. |
| **Communication** | Gives detailed explanations to the questions asked and has correct responses. | Gives detailed explanations to the questions asked and has some correct responses. | Gives explanations to the questions asked but has few correct responses. | Gives explanations to the questions asked but has one or no correct responses. |

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| **Plan Learning Experiences and Instruction** *For each day you should write a short summary of what will be happening that day. These summaries should work together to make it clear how the unit moves forward. You may use bullets to indicate lesson activities, assessments, etc. It is not permissible to just put lecture; instead you should state what the lecture is about (Ex. Lecture on the causes of the Cold War, or interactive discussion about the types of chemical bonds, etc.) Include a time estimate for each activity. If your unit is more than 10 days, you can add additional rows to the calendar. Include the WHERETO elements in the calendar.* |
| **Monday** | **Tuesday** | **Wednesday** | **Thursday** | **Friday** |
| 1* Warm-Up [5 min]
* Students will take notes on trigonometric identities. We will work a few example problems. [20 min]
* Class work [20 min]
* Homework

***(E1, R1, T, O)*** | 2* Warm-Up [5 min]
* Students will take notes on solving trigonometric equations using inverses. We will work a few example problems. Students will be using a graphing calculator during this lesson. [25 min]
* Class work [15 min]
* Homework

***(W1, W2, E1, R1, T, O)*** | 3* Warm-Up – (H1- Students will be told the story of how the acronym SOHCAHTOA started.) [5 min]
* Group work. Students will be split into groups and given instructions on how to use trigonometric ratios to find unknown pieces of a right triangle. The groups will be given 10 problems to complete [40 min]

***(W1, W2, H1, H2, E1, R1, T, O)*** | 4* Warm-Up which will involve problems to prepare students for quiz. [5 min]
* Quiz over sections 1-3 [25 min]
* Students will take notes on area of a triangle. We will work a few example problems [15 min]
* Homework

***(E1, R1, R2, E2, T, O)*** | 5* Warm-Up [5 min]
* Students will be shown the law of sines. They will then be given a worksheet with problems involving law of sines. (H1 – Students will be told that each student that works every problem correctly will receive a bonus point on their test.)

***(W1, W2, H1, H2, E1, R1, R2, E2, T, O)*** |
| 6* Performance Task [45 min]
* Homework over anything not finished in the Performance Task.
 | 7* Warm-Up – (H1 – Students will be shown a video to help them remember law of sines and law of cosines.) [5 min]
* Students will be given a worksheet facedown, and when the teacher says go the students will turn over the worksheet and try to solve their problems correctly first. First student that finishes the most problems correctly gets to write a Law of Cosines problem to put on the test.
* Class work [15 min]
* Homework

***(W1, W2, H1, H2, E1, T, O)*** | 8* Warm-Up [5 min]
* Students will take notes on angle identities. We will work a few example problems. Students will come to work problems on the SMART board. [25 min]
* Class work [15 min]
* Homework

***(E1, T, O)*** | 9* Warm-Up [5 min]
* Students will be given the double-angle identities. We will discuss when to use these identities. After class discussion, students will be split into groups of four, and each group will write a problem in which a double-angle identity would be used to solve it. After each group writes a problem, they will trade problems with another group and solve the other group’s problem.
* Homework

***(W1, W2, E1, T, O)*** | 10* Warm-Up [5 min]
* Students will watch a video on how to solve half-angle identity problems. After viewing the video, students will partner up; each set of partners will solve three half-angle identity problems. After each set of partners has solved their problems, they will come to the board one group at a time and explain how to solve one of their problems to the class.

***(H1, H2, E1, R1, R2, E2, T, O)*** |
| 11* Test Review – Students will have a review competition. They will work problems on the SMART board. [45 min]
 | 12* End of Unit Test [45 min]
 | 13 | 14 | 15 |

***Include WHERETO elements on the Calendar:***

W – Ensure that students understand WHERE **[W1]** the unit is headed and WHY **[W2].**

H – HOOK **[H1]** students in the beginning and HOLD **[H2]** their attention throughout.

E – EQUIP **[E1]** students with necessary experiences, tools, knowledge, and know-how to meet performance goals.

R – Provide students with numerous opportunities to RETHINK **[R1]** big ideas, REFLECT **[R2]** on progress, and REVISE their work.

E – Build in opportunities for students to EVALUATE **[E2]** progress and self-assess

T – Be TAILORED **[T]** to reflect individual talents, interests, styles, and needs.

O – Be ORGANIZED **[O]** to optimize deep understandings as opposed to superficial coverage.

**Lesson 1 – Trigonometric Identities**

Equip: Students will know how to use their knowledge of trigonometric identities to verify trigonometric equations. Students will use this knowledge to

 class work problems and homework problems.

 Rethink: Students will think about trigonometric identities and their relationship to information they have previously learned.

 Tailored:

 Verbal: Students will be involved in class and/or pair discussion.

 Logical: Students will use step by step processes that require thought, to solve problems.

 Intrapersonal: Students will be given the option to work in pairs.

 Interpersonal: Students can work individually on class work problems.

 Organized: Students will be able to verify different trigonometric identities.

**Lesson 2 – Trigonometric Equations using Inverses**

Where: Students will understand that a trigonometric function can have an inverse.

 Why: Understanding trigonometric functions can have an inverse helps students to better understand domain.

 Equip: Students will know how to use their knowledge of trigonometric functions and domain to solve problems involving trigonometric inverses.

 Rethink: Students will reconsider the meaning of domain and how it is different when dealing with trigonometric functions and their inverses.

 Tailored:

 Verbal: Students will be in groups and discussing ideas within their groups.

 Logical:

 Kinesthetic: Students will be moving around when getting in groups and out of groups.

 Visual: Students will be provided with visual representations of trigonometric functions and inverses.

 Intrapersonal: Students will have class work in which they can work in groups.

 Interpersonal: If class work is finished before the end of class, student can work individually on their homework.

 Organized: Students will be able to find the inverses of trigonometric functions.

**Lesson 3 – Trigonometric Ratios**

Where: Students will understand that trigonometric functions can be applied to solve real world problems.

 Why: Understanding how trigonometry is used in the real world will help students realize that mathematics is important.

 Hook: Students will be told the story of how the acronym SOHCAHTOA came about.

 Hold: Students attention should be held during problem solving because they will be trying to remember the acronym.

 Equip: Students will know how to use their knowledge of SOHCAHTOA to remember the trigonometric ratios. Students will solve problems that give them

 real world connections. Students will be split into groups, and each group will be given 10 different problems to solve.

 Rethink: Students will reconsider the importance of trigonometric ratios and how they could apply them in real life.

 Tailored:

 Verbal: Students will be in groups and discussing ideas within their groups.

 Logical: Students will use step by step processes to solve the problems.

 Kinesthetic: Students will be moving around when getting in groups and out of groups.

 Visual: Students will have visuals and draw their own pictures for the problems.

 Intrapersonal: Students will be involved in group work.

 Organized: Students will be able to relate the sides of a triangle by using trigonometric ratios.

**Lesson 4 – Area of a Triangle**

 Equip: Students will know how to use their knowledge of the area of a triangle formula, and knowledge of other mathematical concepts to find the area of a

 triangle.

 Rethink: Students will reconsider other mathematical concepts they have learned and think about how they relate to the area of a triangle and everything

 they have learned.

 Reflect: Students will take a quiz.

 Evaluate: After students have completed and turned in the quiz, we will go over the answers and how to arrive at those answers. Students will be able to

 self-assess how they did.

 Tailored:

 Verbal: Students will be able to participate in class discussion over the quiz and concepts they will learn and revisit after the quiz.

 Logical: Students will have to remember and use steps and concepts when completing their quiz, and use certain processes to complete problems

 from the lesson after the quiz.

 Kinesthetic: Students will be moving around when getting up to turn in their quiz, or getting up to ask questions.

 Visual: Students will have visuals when solving problems on their quiz and problems in the lesson.

 Intrapersonal: Students will be permitted to work in pairs on the class work.

 Interpersonal: Students will be able to work individually on the class work.

 Organized: Students will be able to solve for the area of a triangle.

**Lesson 5 – Law of Sines**

Where: Students will understand that the Law of Sines relates the sines of angles of a triangle to the side lengths.

 Why: Understanding Law of Sines will allow us to perform calculations easier.

 Hook: Students will be told that they do not have to take notes today, they just have to complete a worksheet.

 Hold: Students will be told that the student that completes the most problems correctly will get a bonus point on their unit test.

 Equip: Students will know how to use their knowledge of the Law of Sines to find unknown angles and sides of a triangle.

 Rethink: Students will reconsider what they know about Law of Sines and think about how it relates to other properties they have used to find unknown

 parts of a triangle.

 Reflect: Students will be given back homework where they had to find unknown parts of a triangle and rework the problems that can be solved using Law of

 Sines.

 Evaluate: When students are looking back through homework problems, they will be able to self-asses on what they did, and if and how they could do the

 problems differently.

 Tailored:

 Logical: Students will use new and old processes to solve problems.

 Kinesthetic: Students will be moving around when they get up to turn in their worksheet.

 Visual: Students will have visuals and create visuals to solve problems.

 Intrapersonal: Students will work in groups when they are reworking homework problems using Law of Sines.

 Interpersonal: Students will work individually when doing their worksheet.

 Organized: Students will be able to solve for unknown pieces of a triangle using their previous knowledge and Law of Sines.

**Lesson 6 – Law of Cosines**

Where: Students will understand the Law of Cosines relates the length of a side of any triangle to the measure of the opposite angle and the other two side

 items.

 Why: Understanding Law of Cosines will allow us to perform calculations easier.

 Hook: Students will be shown a video to help them remember the law of sines and law of cosines. https://www.youtube.com/watch?v=-BIfuehcXAE

 Hold: Students will be given a worksheet facedown and told that the first student to correctly solve the most problems will get to write a Law of Cosines

 problem to put on the test.

 Equip: Students will know how to use their knowledge of Law of Cosines to find unknown angles and sides of a triangle.

 Tailored:

 Logical: Students will have to uses step by step processes to solve problems.

 Kinesthetic: Students will be moving around when they get up to turn in their worksheet.

 Visual: Students will have visuals to go along with the problems they will be solving and they will also view a video.

 Interpersonal: Students will work individually on their worksheet.

 Musical: Students will view and listen to a video about Law of Sines and Cosines. The video goes along to the tune of a song.

 Organized: Students will be able to solve for unknown pieces of a triangle using their knowledge of Law of Cosines.

**Lesson 7 – Angle Identities**

 Equip: Students will know how to use their knowledge of angle identities to verify identities and find exact values.

 Tailored:

 Verbal: Students will be able to participate in class discussion when taking notes on angle identities.

 Logical: Students will have to use step by step processes to solve problems.

 Intrapersonal: Students will be permitted to work in groups when doing their class work problems.

 Interpersonal: Students will have the option to work individually when doing their class work problems.

 Organized: Students will be able to find exact values when using their knowledge of angle identities.

**Lesson 8 – Double-Angle Identities**

Where: Students will understand that double-angle identities are special cases for the angle sum identities.

 Why: Understanding double-angle identities will help students to find exact values.

 Equip: Students will know how to use their knowledge of double-angle identities to find exact values.

 Tailored:

 Verbal: Students will participate in a class discussion on double-angle identities.

 Logical: Students will have to use step by step processes to solve problems.

 Kinesthetic: Students will be moving around when getting in and out of their groups.

 Intrapersonal: Students will be working in groups.

 Interpersonal: Students will be able to work individually on their homework if there is time at the end of class.

 Organized: Students will be able to find exact values using their knowledge of double-angle identities.

**Lesson 9 – Half-Angle Identities**

 Hook: Students will watch a video on how to solve half-angle identity problems.

 Hold: Students attention should be held because they will be working in groups, and presenting at the end of class.

 Equip: Students will know how to use their knowledge of half-angle identities to solve problems.

 Rethink: Students will reconsider half-angle identities and how they relate to double-angle identities.

 Reflect: Students will solve a few of their double-angle identity problems using half-angle identities.

 Evaluate: Students will self-assess their methods when they solved the problems using double-angle identities after redoing the problems using half-angle

 identities.

 Tailored:

 Verbal: Students will be able to have discussions within their groups.

 Logical: Students will use step by step processes to solve problems.

 Kinesthetic: Students will be moving around when they are getting in and out of their groups.

 Visual: Students will watch a video on how to solve problems using half-angle identities.

 Intrapersonal: Students will work in groups when doing the class work.

 Interpersonal: Students can work individually on their homework problems.

 Organized: Students will be able to find exact values using their knowledge of half-angle identities.

Notes for the WHERETO elements:

***Once you have completed your calendar you should go through the descriptions and identify the places where you are doing each of the WHERETO elements. You can do this with letters or with the words, as you choose. [W1] [W-2] [H1] or [Where] [Why] [Hook]***

***Keep in mind that some of these elements will be used more than others. For example: Often you will only need to [Hook] the students into the unit at the beginning in most units; however, you can use additional hooks throughout the unit if this is appropriate for your unit and you may want to provide many opportunities in your unit to hold [H2] the students’ attention.***

***You will need to [Equip], [Rethink], [Reflect], and possibly [Revise] throughout the unit.***

***Once you have completed your initial placement of the WHERETO elements on the calendar, you should review what you have planned. If adjustments need to be made to the plan, make these and then redo the WHERETO elements as appropriate.***