New approaches to algebra: Have we missed the point?

Thornton, S. J. (2001). New approaches to algebra: Have we missed the point? Mathematics Teaching in the Middle School, 6(7), 388-392. Retrieved from http://search.proquest.com/docview/231301171?accountid=11662

Abstract (summary)

Thornton looks at three approaches to algebra: a patterns approach, in which students are aksed to generalize a relationship, a symbolic approach, in which students learn to manipulate algebraic expressions, and a functions approach, which emphasizes generation and interpretation of graphs.

Full Text

CURRICULUM MOVEMENTS IN THE

United States and Australia, characterized by such documents as Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) and A National Statement on Mathematics for Australian Schools (AEC 1991), have challenged the conventional view of algebra as formal structure, arguing that algebra is fundamentally the study of patterns and relationships. Increased emphasis has been given to developing an understanding of variables, expressions, and equations and to presenting informal methods of solving equations. The emphasis on symbol manipulation and on drill and practice in solving equations has decreased (NCTM 1989).

Has the net effect of these changes been merely to replace one kind of procedural knowledge with another? This article looks at three approaches to algebra: (1) a patterns approach, in which students are asked to generalize a relationship; (2) a symbolic approach, in which students learn to manipulate algebraic expressions; and (3) a functions approach, which emphasizes generation and interpretation of graphs: This article examines the nature of thinking inherent in each approach and asks whether any or all of these approaches are, in themselves, sufficient to generate powerful algebraic reasoning.

The Patterns Approach, or "Matchstick Algebra"

THE PATTERNS APPROACH TO ALGEBRA IN THE middle school is typified by the matchstick pattern shown in figure 1. Faced with this problem, students almost invariably describe the rule as "add 3." Most students look at the table of values horizontally, observing that each time a square is added, the number of matches needed increases by three. Well-intentioned teachers often help students find a general rule from this observation, saying, for example, that if one adds 3 each time, the rule is of the form $m = 3s + k$, and suggesting that students try a few numbers to determine the value of the constant.

The students regard this approach as good teaching because it helps them obtain the correct answer. The teacher is similarly reinforced in the belief that he or she is acting in the students' best interests, because the students are able to find the rule for this pattern and, perhaps, even a general rule for other linear cases. The ability to find these rules is, arguably, a useful skill, but do the students understand any more about the nature of algebra than if the subject had been introduced in a formal, symbolic way? Students who use this heuristic to find the constant and thus the general rule have, in reality, looked at the specific rather than the general. They have not necessarily acquired any welldeveloped notion of the general nature of the pattern but have merely learned a procedure to develop a correct symbolic expression. The algebraic essence of the problem is absent

The Matchstick Pattern Problem is not about finding a general rule. The answer to the problem, that is, the rule itself, is unimportant. The problem is really about alternative representations. It is a visualization exercise in which different ways of looking at the pattern produce different expressions. Visualizing the pattern in different ways and writing corresponding algebraic relationships help students understand the nature of a variable and become familiar with the structure of algebraic expressions. This particular pattern can be visualized in at least four different ways (see fig. 2).

Writing down the number pattern in a table, an activity commonly found in textbooks and on worksheets, does not help students visualize the generality inherent in the matchstick constructions. A much more constructive approach is to ask students to build one element of the pattern physically and explain how it is put together, not in terms of numbers but in terms of its underlying physical structure. The different algebraic structures then have direct physical meanings.

Numerous other visual approaches to algebra are possible (Nelsen 1993). For example, students could be asked to visualize the pattern shown in figure 3 in different ways so as to generate a relationship between the number of shaded squares (b) and the length of the side of the white square (n). Again, at least four different representations are possible (see fig. 4). The point of the exercise is not to obtain the answer $b = 4n + 4$ or any of its variants but rather to understand how the pattern can be visualized and how these different visualizations can be described symbolically. If we are to foster powerful algebraic thinking in our students, we must encourage a variety of well-justified generalizations of the pattern. Rather than be an end in itself, the purpose of generating rules is to develop insight into patterns and relationships. As Gardner (1973, p. 114) writes, "There is no more effective aid in understanding certain algebraic identities than a good diagram. One should, of course, know how to manipulate algebraic symbols to obtain proofs, but in many cases a dull proof can be supplemented by a geometric analogue so simple and beautiful that the truth of a theorem is almost seen at a glance."

The Symbolic Approach, or "Fruit Salad Algebra"

THE FORMAL, SYMBOLIC approach to algebra, in which variables are defined as letters that stand for numbers, has been criticized as lacking meaning (Chalouh and Herscovics 1988) and has been identified as the source of many difficulties faced by beginning algebra students (Booth 1988). Olivier (1984) used the term "fruit salad algebra" to describe an approach to algebra in which students choose variables as objects or labels rather than as numbers. The fruit-salad approach to algebra is illustrated by such questions as the following (Haese et al. 1991, p. 198):

Three people have two apples and a banana each and two other people have an apple and three bananas each.

(1) How many apples and bananas did they have altogether?

(2) Expand and simplify 3 (2a + b) + 2 (a + 3b) .

(3) What do you notice about (1) and (2)?

This approach provides concrete models for symbols, which can lead students to short-term success, but the confusion of variables with objects can also lead to widespread misunderstanding (MacGregor 1986). To help alleviate this confusion, some much more effective and powerful models, such as algebra tiles, have been developed to give concrete meaning to symbolic manipulation.

No matter how effective the model or context is, a potential danger is that the fundamental meaning and purpose of symbolic manipulation may be lost. The essence of symbolic manipulation lies not in obtaining an answer to a specific question but rather in helping make sense of an observation. Each different representation produced through symbolic manipulation can reveal a different insight into the situation being considered. For example, Crossfield (1997) describes using colored numbers in his classroom as a strategy for helping students develop general results. His students observe, for example, that if two square numbers are subtracted, the result can only be a prime number if the two squares are consecutive squares, and even then only if the sum of their square roots is a prime number. In every other case, a composite number results. The result that a^sup 2^ - b^sup 2^ = (a + b) (a - b) shows that if a - b # 1, the result cannot be prime.

Calendar patterns (Olssen 1995) also present an ideal context in which to emphasize the meaning and purpose of symbolic manipulation. In one exercise, a square of four numbers is selected at random from any month of a calendar (see fig. 5). Multiplying the two shaded numbers along each diagonal in figure 5 and then subtracting their products produces the result 10 x 16 - 9 x 17 = 7. Every square of four numbers, taken from any month, will result in a difference of products equaling 7. As it does in the exercise with colored numbers, the algebraic justification for this observation lends meaning and purpose to the processes of symbolic manipulation. One of the most significant features of this problem is the position of each number in the calendar in relation to the others (see fig. 6). Thus, d + 1 and d + 7 have not only a numerical significance but also a positional significance. Identifying algebraic symbols both numerically and visually enhances students' algebraic thinking. An almost endless supply of similar results can be found embedded in a simple calendar.

Symbolic manipulation has a purpose. The search for concrete models in algebra should not mask the fundamental purpose of symbolic manipulation as communicating insight into generality by enabling an expression to be written in different ways. Each different form of an expression should reveal a different feature, leading to the discovery, communication, and proof of general results.

The Functions Approach, or "Taxicab Algebra" all.

THE FUNCTIONS-AND-GRAPH APPROACH TO algebra is characterized by the problem shown in figure 7. This approach emphasizes a realistic application of graphs and encourages students to represent situations in words, symbols, graphs, and tables. Yet seldom do teachers or textbooks ask students to discuss the different insights that can be gained from each of these representations.

The four representations lead to twelve possible transitions between representations, as shown in figure 8. By far, the most common path around this diagram is counterclockwise, moving from words to numbers to graphs to symbols. Seldom do we expect students to move directly from words to symbols or from tables to words. Yet all twelve of the possible transitions add to students' understanding of the nature of functions and relationships. For example, the ability to recognize that the relationship in the Taxicab Problem can be written as c = 180 + 120k, without having to draw a graph or a table of values, shows insight into the relationship between cost and distance. Understanding why this symbolic representation is useful-because it makes the problem easier to solve using a spreadsheet or programmable calculator or to work the question backward-shows an even greater degree of insight into the situation itself and into the power of algebraic thinking. The ability to represent a relationship graphically, in a table, as a function, and in words, and the thinking required to convert directly from one representation to another in every possible permutation is to promote the development of different insights into the situation being studied.

Conclusion

THE REFORM OF SCHOOL MATHEMATICS CURricula has tended to deemphasize the formal, symbolic approach to algebra and emphasize patterns and functions as central themes of middle school mathematics (NCTM 1989). Clearly, all three approaches-patterns, symbols, and function-are of central importance; however, an unquestioning acceptance of these approaches to algebra, even as outlined in current curriculum documents, can still fail to develop powerful algebraic thinking in our students. We must examine the fundamental nature of algebra before we can help our students use algebra for a purpose. The power of algebra lies in its capacity to develop and communicate insight by representing situations in alternative ways. Whether developed through alternative visualizations, symbolic manipulation, or a functional approach, each of these alternative representations leads to new insights into mathematical relationships. As teachers of algebra, our role is to help students take advantage of the insights illuminated by these alternative representations.

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