Factor Game

(Number Theory)

Objective

Students will use a game setting to identify the properties of prime, composite, abundant, deficient and perfect numbers.

Overview of the Lesson

Using a Factor Game Board, comprised of the numbers from 1 to 30 placed in a 5 by 6 grid, the teacher challenges students to a game. The rules are explained as the game progresses. Basically, Player A circles any number and, using a different color, Player B circles all the factors of that number. Once a number is circled, it cannot be used again. The game continues with Player B picking a number and so on. If a player picks a number that has no available factor, the player loses a turn. The winner is determined by adding the numbers circled by each player. The player with the highest total wins. After playing a game between the teacher and the class, students are given game boards and play against each other. The game is analyzed by filling in a table of first moves. On the basis of the data, students rate each number as a "good" or "not good" first move. Students learn that they can score more points by picking certain numbers. Perfect, deficient, and abundant numbers are described and defined. Students then play the game using the strategies they discovered.

Materials

❶ Transparency: Factor Game Board (or a copy of game board drawn on the board)

❷ Worksheet: Factor Game Board

❸ Worksheet: Analysis of First Moves

❹ Colored pencils

❺ Calculators

Procedure

Project a transparency of a Factor Game Board on the screen. (If an overhead projector is not available, draw a 5 by 6 grid containing the numbers from 1 to 30 on the board.). Use two different colors or two different symbols to distinguish the moves made by the two opposing teams. In the video, the teacher used circles and squares.

Create a friendly opening for your students by informing them that they will be playing a game. The object of the game is to obtain more points than your opponent. In the first game, the teacher challenges the entire class. Inform the students that the rules of the game will be revealed as needed while the game is in play.

To begin, the teacher will be Player A and the class will be Player B. (See **Rules of the Factor Game** below.) Start the game by circling one of the numbers on the game board. After you have circled your number, tell the students to identify all of the factors of that number and circle them in a different color. Scores can be subtotaled as each opponent earns points while the game is in play or students cans be in suspense until the game is over at which time the points are totaled.

Rules of the Factor Game

- Player A selects a number from the Factor Game Board and circles it with his/her colored pencil. Player B then finds all of the factors of that number and circles them with a different colored pencil.
- Next, Player B selects and circles a number from the game board. Player A then finds all of the available factors of this number and circles them.
- Play continues until there are no more numbers which have available factors left on the board.
- **Illegal Move (Penalty Move):** If a player chooses a number that has no available factors, this is called an illegal move. While the player gains those points, the next turn is lost. As a result, the opponent selects the starting number, two turns in a row. Electing to make an illegal move could be considered a winning strategy move. If only illegal moves are possible, this signals the end of the game and no more points are awarded.
- Each should total all of the numbers that are circled in their respective color. The student with the highest total is declared the winner.

Following the initial teacher-student game, place students in groups of two or four to play each other. Provide each group with several factor game boards and two different colored pencils. Refer to the rules above if questions emerge while the games are being played. Allow students time to play the game several times.

Next, have students begin to formally analyze the Factor Game by posing the following questions:

- Which is the best possible first more?
- Which is the worst possible first move?

So that students will have a basis from which to respond to these questions, either provide a chart or have them construct a chart to analyze the "what if's" regarding first moves. Have students categorize those moves which are "good" moves. All moves that provides a player with more points than his/her opponent is considered to be a *good* first move. For example, if Player A's first move is a 5, then Player A's score is 5 and Player B's score is 1, because 1 is the only factor of 5 that is not circled on this game board. This is considered to be a *good* first move.

Discuss questions similar to the following:

- What are all of the first moves which will allow your opponent to only score 1 point? *(These numbers are called **prime** numbers.)*
- Are all prime numbers good first moves?
- What seems to make prime numbers special?
- What are the first moves that will allow your opponent to score more than 1 point. (These number are called ***composite*** numbers.)
- Are composite numbers good first moves? Explain why or why not?
- Are perfect numbers good first moves? Why or why not?

As students classify first moves as *good* or *not good*, the possibilities for the sums of the proper factors of numbers will emerge. There are three possibilities for these sums: they can be larger then the number, smaller than the number or equal to the number. Mathematicians used these patterns to classify numbers as either **abundant**, **deficient**, or **perfect numbers** respectively.

Allow them to play the game at least one more time as they apply their strategies in order to win.

Extensions & Connections

Play a Factor Game Team Tournament.

- Place students in teams of four or five.
- Provide time for each team to select a name, create a logo, and to plan, and counter plan their strategies for being the Factor Game Class Champion Team.

☛ Devise a tournament plan and schedule. (This will be a good activity for the students.) Decide how teams will compete.

☛ Facilitate the tournament until a winning team is declared.

Long ago, astronomers observed the repetitive action of the sun rising and setting at somewhat equal intervals. They decided to use the amount of time between two sunrises as the length of a day. They divided the day into 24 hours. The standard units of time are the second, the minute and the hour. Use what you know about factors to answer the following questions about these units.

- Why is 24 a more convenient choice for the number of hours in a day than 23 or 25?
- If you were to select another number to represent the hours in a day, what would be a good choice? Why?
- Astronomers divided an hour into 60 minutes. Why is 60 a good choice compared to 59, 61 or 45?
- If you were to select another number to represent the minutes in an hour, what would be a good choice? Why?

Have students construct a 40-Factor Game Board. They should investigate good and bad first moves as they play the game. Instruct them to write a summary of their findings.

Resources

National Science Foundation. Connected Mathematics Project. *Prime Time: Exploring Factors and Multiples.*

Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

Standard 1: Worthwhile Mathematical Tasks

❶ In order to develop students' mathematical skills, we often begin by stating the objective. In this lesson, the objective was never stated but was developed during the activity. In your opinion, what are the advantages and disadvantages in using this approach?

Standard 5: Learning Environment

❷ It is a key function of the teacher to develop and nurture students' abilities to learn with and from each other. . ." (p. 58) The video teacher used a game to help students learn from each other. How do you set this climate in your classroom?

Standard 6: Analysis of Teaching and Learning

❸ "Student journals are yet another source that can help teachers appraise their students' development."(p. 64) Twice during the lesson, the video teacher instructed her students to write in their journals. The first time was to describe their game playing strategy before the analysis and the second time was to describe their strategy after the analysis. Do you think that maintaining student journals is an effective way to assess students' conceptual and procedural understanding in the mathematics instructional program? Why or Why not? Should it be optional for students to share their journal entries with the teacher/class?

❹ "Students' disposition toward mathematics — their confidence, interest, enjoyment, and perseverance — are yet another key dimension that teachers should monitor." (p. 63) Suppose one of your students has lost every game that he has played, even after analyzing the first moves of the game. Frustration sets in and his once positive disposition changes to a very negative one. How would you solve this problem?

❺ Students were almost tricked into learning new terminology relating to some number theory concepts. Do you think that this game playing environment provided a sound context for learning these concepts or do you view this learning as basically superficial and short-lived? Why do you feel this way?

❻ If you use this lesson with your students, what is the single most important outcome that you would want every student in your class to be able to demonstrate?

The Factor Game Board

1	**2**	3	4	5
6	**7**	8	9	10
11	**12**	13	14	15
16	17	**18**	19	**20**
21	22	**23**	24	**25**
26	27	28	29	30

Analyzing First Moves for the 30-Game Board

1st No. Picked Is:	Opponent Gets	SUM	Factors GOOD	Factors NOT GOOD
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
12				
13				
14				
15				

	Factors		Factors	
1st No. Picked Is:	Opponent Gets	SUM	GOOD	NOT GOOD
16				
17				
18				
19				
20				
21				
22				
23				
24				
25				
26				
27				
28				
29				
30				