CASE 5

Merseth, K. K. (2003). Case 5 - Seeing is Believing. In *Windows on Teaching Math: Cases of Middle and Secondary Classrooms*. (pp. 37-41). New York: Teachers College Press.

Seeing Is Believing

PRE-CASE EXERCISES

Before doing these exercises, you should clear the graphing window of your TI–82 (or similar) calculator. If you need assistance in doing this, refer to the following instructions:

1. Press [MODE] and select [Normal, Float, Radian, Func, Connected, Sequential, and FullScreen] by using the down arrow to highlight each term and pressing [ENTER].

   Return to the home screen by pressing [QUIT]. Notice that [QUIT] is a [2nd] function located above [MODE].
2. Press [Y=]. Position the blinking cursor to the right of each active "=" sign and press [CLEAR]. Return to the home screen by pressing [QUIT].
3. Press [STAT PLOT], use the down arrow to select [4:PLOTS OFF], press [ENTER] once, and then press [ENTER] again. Notice that [STAT PLOT] is a [2nd] function located above [Y=].

   Alternatively, you can press [STAT PLOT], press [4], and press [ENTER] once.

   Whichever way you choose to do step 3, it is essential that you see the word "Done" on the right side of your home screen. In the first method, "Done" appears after you press [ENTER] the first time, see "PlotsOff" on the home screen, and press [ENTER] again.

   In the second method, you should see "PlotsOff" on the home screen after you press [4]. You should see "Done" after you press [ENTER].

   If you forget to press [ENTER] and do not see the word "Done" you will eventually get an error message. If you do get an error message, select [2:Quit] and repeat step 3.
4. Press [DRAW] (located above [PRGM]), select [1: ClrDraw]; and press [ENTER] twice. Again, be sure you see the word "Done" on the home screen.
5. Press [WINDOW], use the right arrow to select [FORMAT]. Select [Time, RectGC, CoordOn, GridOff, AxesOn, and LabelOff].

You have completed the steps to clear the graphing window. In addition, the calculator used by Megan and Mr. Wenmark in the following case was preset with a zoom factor of 4. Check the zoom factor currently set on your calculator by doing the following. Press [ZOOM], use the right arrow key to select [MEMORY], select [4:SetFactors . . .], and press [ENTER]. If [XFact] and [YFact] are not already set to "4," change their current settings by typing in the number "4." You are now ready to work on the following problem, which is the focus of the case.

Using your calculator, draw an appropriate graph and use it to find the roots of the equation: $x^2 - 49.5x + 612 = 0$.

1. Provide a solution to the problem that includes a description of how you solved the problem.
2. The given equation is in the general form, $ax^2 + bx + c = 0$. Determine a value for "b" and "c" that would give an equation with one root? No roots? Explain how you arrived at your answer. What general observations can you make at this point?
3. What is the role of estimation in solving this problem? If you were a teacher would you be satisfied with an estimate? Explain you answer.

THE CASE

"Mr. Wenmark, I'm really lost on this graphing calculator stuff," complained a frustrated Megan Farrell, as she paused at the end of class.

"You are?" replied Paul Wenmark, a little puzzled. "You're usually one of the best students in here. What's the problem?"

"I just don't get it," she continued. "Sometimes I see answers on the screen that you say are wrong, other times, I don't see anything. I really don't understand how this thing works."

"How can that be?" Mr. Wenmark asked.

"Now Mr. Wenmark, if I knew that I wouldn't be here now," Megan replied with a smile. "I just don't get this stuff. I really need help before tomorrow's quiz."

Earlier in the day, Mr. Wenmark had worked with his class of accelerated Algebra II sophomores where the students were using graphing calculators in groups to find the roots of quadratic equations. He felt the class had gone well. He ended the class by announcing that there would be a quiz the next day on finding roots with the graphing calculators. Megan approached him after class.

"Can you come after school?" Mr. Wenmark suggested. "I'll be here for about fifteen spare minutes, then I have to go to the principal's meeting."

"You know I have basketball practice," she scowled. "Ms. Smith gets very annoyed if we are late. Can't we go over it now? You have cafeteria study and you could write me a note to be late for English."

"No, no, no," he said, equally annoyed. "We can't do that, you can't miss another class! Just come here after school."

Megan pouted. The bell rang. Mr. Wenmark scrawled her a quick late note and they both hurried off.

Mr. Wenmark liked Megan. She was an alert, eager math student. Usually her grades were As or high Bs. She was in the top 10% of the sophomore class and a pretty good basketball player. He was very surprised that she was having trouble with the graphing calculator.

Mr. Wenmark and Graphing Calculators

Mr. Wenmark had been teaching at Chapman High, a suburban high school of 1,500 students, for 6 years. He taught Business Math, Algebra I, and Algebra II. Recently, the math department had expanded their supply of graphing calculators, and the department had agreed to use graphing calculators in Algebra II. The following year they were planning to introduce graphing calculators in Algebra I.

For the last 2 years, Mr. Wenmark had been hoping to use graphing calculators, since he was always hearing how important technology was, but the department didn't have enough money to purchase a sufficient number of class sets. "Besides," he thought, "maybe I can learn by using them in Algebra Two rather than going to some stupid workshop."

Mr. Wenmark was, however, rather intimidated by the graphing calculator. He had played around with one now and then but barely felt comfortable with the machine. Clearly he had never used one as a high school student. And he was uncertain how much it would change the curriculum and his teaching, with his lack of experience. Furthermore, he had no feel for the types of errors his students might make.

His cafeteria duty that day was quiet, so he had time to reflect on the way he introduced calculators to his Algebra Two class. He had started using the calculator when they were graphing lines. After graphing lines by hand, he taught them how to do it with the calculator. The students were amazed at how much easier it was to do on the calculator. The more vocal students pressed him to explain why he had them graph by hand in the first place. "Why can't we just use the calculator all the time, Mr. Wenmark?" Alicia Fagan had asked. As he thought about the class, he did not remember Megan expressing any difficulties. He looked up her grades. His grade book showed she received an 86 on the quiz using the graphing calculator with linear equations.

The class had just started work with conics. During the previous week, they had been finding the roots using the quadratic formula. Megan got a 92 on that quiz. Mindful of the student objections he had received when they did the line unit, Mr. Wenmark did not want to just repeat material with the calculator. He was, therefore, covering the section on finding the roots graphically by only using the calculator. He wondered what Megan's problem was.

After School

Megan came back to Mr. Wentmark's room after school. "Hi, Megan. Let's get right to work.

We're both in a hurry. Can you show me what you're having trouble with?"

"I got how to graph a line, sort of. But now I get either parabolas or lines." Mr. Wenmark gave a mystified look. "I mean, sometimes you say the graph should be a parabola, but I get a line. Before, all I could get were lines and that was all I ever wanted." Megan drew a line and a parabola in the air.

"Are you changing the scales, so you can see the whole curve?" asked Mr Wenmark.

"What do you mean?" queried Megan. "How do I know if I should do that? I mean, look, here's one you had us do in our groups today: $x^2 - 49.5x + 612 = 0$. Watch, I'll put it in and use ZStandard [zoom standard] for my viewing window range. See, I get a line" (see Figure 5.1).

"But Megan, you know it's a parabola," Mr. Wenmark said in a slightly irritated tone. "The x is squared. It couldn't be a line."

"But Mr. Wenmark, how would I graph something if I didn't already know what it looked like?" said Megan, beginning to get frustrated herself. "It reminds me of trying to find a word in a dictionary if you don't know how it's spelled. I could never find something in a dictionary unless I already knew how to spell it. Now, every time I use a graphing calculator to graph something, do I have to know what it looks like before I begin?"

"For tomorrow's quiz, there are only going to be parabolas," Mr. Wenmark said as he glanced at his watch. "Later we'll do different curves. Let's get used to the machine first. Do you know how to widen the window so you can see the whole graph?"

Figure 5.1 Megan's Graphing Calculator

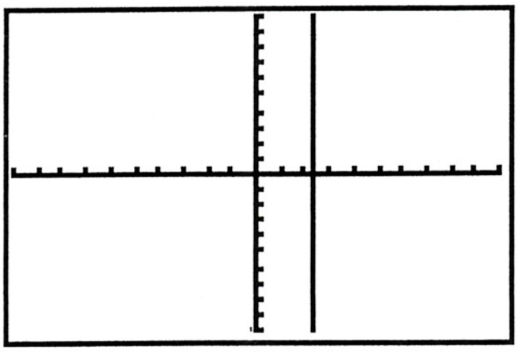
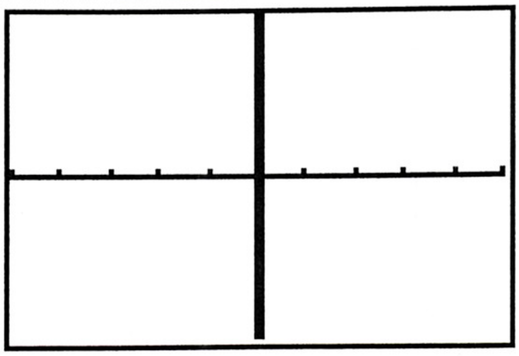


Figure 5.2 Megan Tries Again



"Do I change the window format?" Megan asked as Mr. Wenmark nodded. "So, I press [GRAPH] and then [WINDOW]. Now I do the Xmin and the Xmax."

"Right, change them to, say, Xmin = –100, Xmax = 100, Xscl = 20, Ymin = –100, Ymax = 100, and Yscl = 20," he said quickly. "Then we should see the parabola."

Megan pressed the buttons, stared at the screen and wrinkled her nose. "Mr. Wenmark, look at this. It's a really skinny line right next to the y axis" (see Figure 5.2).

"*What*? Where are the two parts of the parabola? It can't be a line!" Mr Wenmark said with surprise. Megan just stared at him. "Let's see what you put in for the function." He grabbed the calculator out of her hand. Pressing Y=, he saw: $y_1 = x^2$ -49.5x + 612. He squinted at the screen. "Ah, you see this -49.5? The minus sign is too short. You pressed the negative button instead of the subtraction button. Remember, I warned the class about that. What did you graph? You graphed a cubic, not a line."

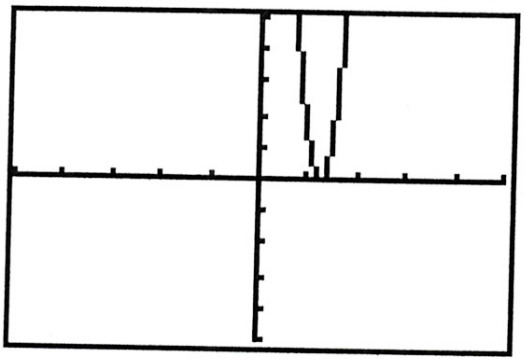
"A what?" asked Megan hesitantly.

"You graphed negative 49.5 times x cubed plus 612," he replied. "Now, come closer. Watch me change this minus by pressing the subtraction button and we'll get a parabola" (see Figure 5.3).

"Thanks, Mr. Wenmark," responded Megan. "That looks like one root, around 25.5."

Mr. Wenmark went to the board and expanded $(x - 25.5)^2$ obtaining $x^2 - 51x + 650.25$. "That doesn't match the equation you entered, so it can't be 25.5." He then looked at the clock. "I know there are two roots. Look, Megan, I'm real late to

Figure 5.3 Mr. Wenmark Demonstrates



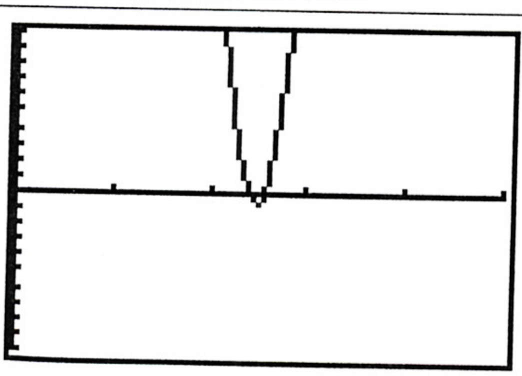
see Mr. Yutzy. It's about one of the kids in your class. I'll only be gone for five minutes. You stay here and get the roots on the calculator."

"But, my basketball!" fumed Megan. Mr. Wenmark did not reply as he rushed out the door.

After contemplating leaving, Megan picked up the calculator, pressed the WINDOW button, and changed the viewing window scale to [0, 50, 10, -10, 10, 1]. After she pressed [GRAPH], the graph in Figure 5.4 appeared.

Damn, he was right—there are two roots, Megan said to herself. After tracing the curve, Megan wrote down, "The roots are 23.93617 and .09981893." Megan looked at her paper for a while before realizing what she had done. "Wait a minute," she said, "something's wrong here. *Y* should be 0. I have the *x* value for $y = .09981893$ and called the *y* value a root. What do I do now?" Perplexed, Megan stared at her calculator and absent-mindedly traced to the other place where the parabola crossed the axis. The cursor values at that point were $x = 25.531915$ and $y = .0488909$. "The *y* value still isn't 0," Megan said.

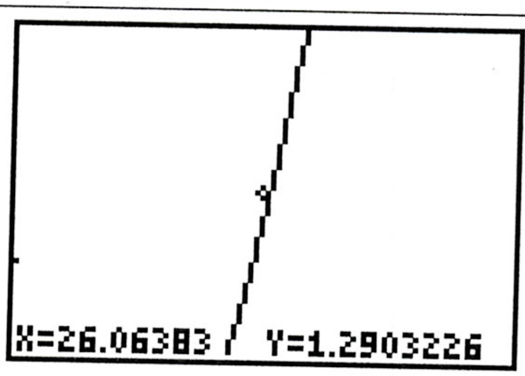
Figure 5.4 Megan Finds Two Roots



She pressed the right arrow to move the cursor along the function to the next plotted point. The coordinates of that new plotted point were $x = 26.06383$ and $y = 1.1636487$. "That didn't help, maybe I'll try [ZOOM]," Megan said. Leaving the cursor at the same plotted point, she pressed ZOOM and then "2" for [ZOOM IN] and [ENTER]. Wanting to see a more magnified picture on the viewing window, she zoomed in one more time. The graph shown in Figure 5.5 appeared.

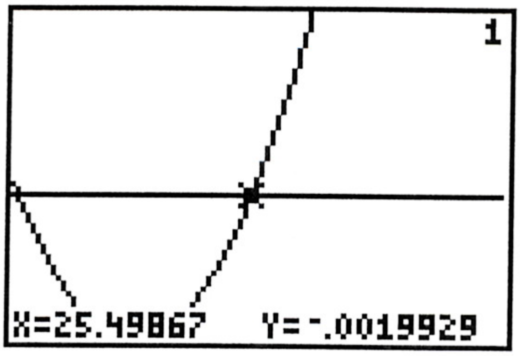
Just then, Mr. Wenmark rushed in from his meeting looking a bit flustered. When Megan saw him, she slammed the calculator on the desk and yelled, "Why am I missing basketball for this stupid piece of technology? I don't need this! I'm going to be a sports writer anyway." Megan got up to leave.

Figure 5.5 Megan Zooms In



He put his hand on her arm. "Hey, wait a minute." Mr Wenmark took her calculator. "I don't know how you got this graph. But here, look, watch how I do it. Let me show you." He reset the viewing window to [0, 50, 10, -10, 10, 1] and graphed the equation. He briefly traced around the curve and left the cursor on the plot ted point with coordinates $x = 25.531915$ and $y = .0488909$ and zoomed in. The coordinates of the plotted point were $x = 25.531915$ and $y = 0$. Mr. Wenmark stared at the viewing window and wondered how that happened. He traced along the curve a few times, but could not get $y = 0$ again. Deciding to worry about that issue later, he placed the cursor at $x = 25.531915$ and $y = .0488909$ and zoomed in again (see Figure 5.6).

Figure 5.6 Mr. Wenmark Traces the Curve



Mr. Wenmark then traced along the curve briefly, leaving the cursor on the plotted point x = 25.49867 and y = –.0019929. "Megan, notice that the next plotted point above the x axis is x = 25.531915 and y = .0488909. The second decimal place changes when I press the left arrow and change the cursor from the point just below the x axis to the one just above the axis." Looking at his watch, he continued, "Now you remember how to use the [ZOOM] function, don't you? To get a root accurate to two decimal places, you just have to keep zooming in until there's no change in the second decimal place of the y value when you switch the cursor between points just below and just above the x axis. I've gotta go. My job at the video store starts in 15 minutes, and I haven't had a thing to eat since 11:00. Try it at home. You know enough for tomorrow's quiz. Here's a note for your coach." He ignored Megan's scowl as he rushed out the door.

QUESTIONS

1. What does Megan understand about the roots of quadratic equations?
2. What are some of the factors teachers should consider when working with calculators in the classroom?
3. Is it possible that the use of technology in mathematics classes can be more confusing than clarifying to students?