CASE 2

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The Marble Line

PRE-CASE EXERCISES

Please consider the following experiment (entitled "The Raven"), which is intended to strengthen student understanding of linear functions.

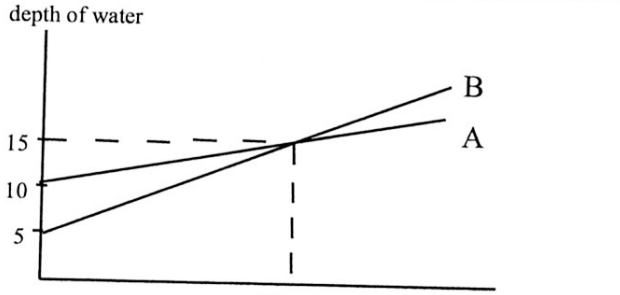
Essentially, the experiment involved starting with a clear cylindrical vessel, pouring in an arbitrary amount of water, depositing marbles of uniform size into the container, measuring the height of the water level, and plotting the height as a function of the number of marbles in the container.

1. Suppose for a particular version of the Raven experiment, one marble gives a height of 12.3 cm and five marbles give a height of 15.3 cm.
   a. What is the height of the water in a cylinder with no marbles?
   b. How much does the depth of the water rise when one marble is added?
   c. Write an equation relating the depth of the water, d, to the number of marbles, n.

2. Sketch graphs of the following equations with x and y both integers.
   a. $y = 2x + 1$
   b. $2x + 3y = 24$

3. Suppose sand is poured into a cylindrical beaker containing some water. The depth of the water depends on the quantity of sand that has been poured in. The graph in Figure 2.1 shows the results for two different beakers, A and B.
   a. What is the depth of the water in each cylinder with no sand in it?
   b. How much does the water rise in each cylinder when 1 gram of sand is put in it?
   c. Which cylinder has a smaller diameter?

Figure 2.1 The Raven



THE CASE

The management team's strategy appeared to be working beyond everyone's expectations. In just a few months the new principal, who was appointed in June and worked all summer, had his fingerprints on myriad projects, from granting flexible scheduling to teams to bedecking the halls with students' academic projects and artwork. And staff learning seemed almost as important as student learning. The principal practiced his belief that teachers armed with adequate knowledge and support can make all the difference in the world and so was eager to create opportunities for intensive staff training in content and instructional strategies connected to cognitive research and adolescent development.

Early Warning Signals

Now it was October, and Jack Mann, the principal of West Middle School, picked up the phone when his secretary signaled a call for him. Alison Tansey, the assistant superintendent for curriculum and instruction in the Barberton Public

Schools, was on the other end. Alison had called, wanting Jack's support to ensure that the math teachers at West would attend the professional development workshops that she was planning. Any teacher's absence at this early stage could signal to others that there was an exit option, which Alison definitely wanted to avoid. One teacher, Sally Elmore, had already contacted Alison and said that she wasn't sure she had the time to attend the workshops.

"Hiya, Al," Jack said. He waited for her to proceed.

"Hey, Jack," Alison responded. "Do you remember Sally Elmore?"

"Oh, sure," Jack replied. "Mrs. Elmore's one of those kind, sweet elementary teachers who moved up here this fall. Nice lady who has a lovely encouraging way with the kids. In fact, just yesterday, I visited her classroom for about 15 minutes. It's wild how I can see the elementary school background of some teachers here. Anyway, Al, it isn't all rosy here in the Land of Oz. I had a flash of déjà vu, watching her. Déjà vu from that video we saw last year—what was that thing called?"

"The TIMSS tape?" Alison offered. TIMSS stands for the Third International Mathematics and Science Study, the largest and most comprehensive study of math and science education ever conducted, spanning 41 countries. Earlier, district administrators had viewed a videotape depicting typical TIMSS classrooms in Japan and in the United States.

"Yeah, TIMSS, that's it," Jack said. "She just didn't seem all that comfortable with the math. She was fine with the kids, but the math . . . ah, that's something else. She just accepted whatever the kids said without asking for explanations. And with the students who had wrong answers, well, she said something like, 'Not quite,' and moved on to another student." Just then, Alison could hear the piercing sound of the fire alarm. "Hey Al, gotta go, can we talk later?" Alison barely got out a reply before the phone went dead.

West Middle School

West Middle School, one of the six middle schools in the district, had some 1,200 students and 80 classroom teachers. It was under a spotlight for rather lackluster test scores, higher than average teacher and student absentee figures, and significant staff turnover. The prior spring, the district management team had moved several principals to different schools, primarily to invigorate West. Jack Mann was assigned to West with a mandate from Dr. Barbara Gross, the Barberton Superintendent, to "shape the place up." Jack Mann had worked in the district for 10 years as an elementary school principal and was well regarded as a manager who could "make the trains run on time." His first priority was making sure his building was operating smoothly, but a close second was to lead his school into the "21st century." He and Superintendent Gross wanted to keep ideas percolating in the school. Jack was the kind of principal who never met a good idea he didn't want to immediately implement in his school.

Professional Development for Mrs. Elmore

Sally Elmore pushed open the doors to the conference room. The teachers were seated at one large table, listening attentively to a single speaker. Her heart sank, thinking she must have missed the beginning of the session. She heard Nicole Wittershein exclaim, "But that's not the most serious problem!"

Nicole went on and on about the TABS test, the "Test of Abilities of Basic Skills" that the district asked middle school teachers to administer each spring in order to support placement decisions for the following year. Though Mrs. Elmore was new to the middle school, she had heard Nicole's plaint before. It had to do with students' woeful computation skills. The kids were never taught their math facts in elementary grades, the story went. Mrs. Elmore was amused by the inevitable slip between "teaching" and "learning" in such stories. Of course the elementary teachers had taught students their math facts. She could even attest that the students had learned them, but more frequently than she wanted to admit, it was clear that the students did not remember what they had learned just a few months before.

By the end of the second hour of the workshop session, Mrs. Elmore had experienced a huge range of emotions. She was exhilarated by the opportunity to use the graphing calculator, but grew terribly frustrated when she would try to have the calculator draw the graph of a linear equation and the screen was blank. She had no idea why this was happening.

Late in the session, the workshop leader fixed on Iris Fox, a new teacher whom the leader had asked to offer a rule for the sequence: 2, 6, 10, 14, 18 . . . Iris wrote $x + 4$ on her paper. Iris was shocked when the leader challenged this answer. He asked her what the 23rd number in the sequence would be and to explain her thinking to the class, emphatically adding that it was *not* 27. As she stammered, it became clear that Iris had no idea what to do. "Think about it," he implored her. Mrs. Elmore was so uncomfortable watching her colleague that she did not follow Iris's explanation when she finally discovered the formula.

At lunchtime, Sally Elmore approached Iris in the buffet lunch line and asked her, "Are you okay?" Sally pretended to focus on the display of tuna, turkey, and veggie roll-ups in front of her.

"Well yeah, sure," Iris responded. "And how about you?" Iris returned the inquiry.

"Oh, me? I'm fine." This was a patent lie, as Sally was getting knots in her stomach from feeling lost in the content and from the anxiety that the workshop leader would call on her. "I was just wondering how you're liking the class."

"I think it's great!" Iris said. "I mean, he's tough, but good. I feel like I'm learning a lot. Nothing like being on the hot seat to activate the gray matter!" Iris laughed heartily, then moved away to join a first-year teacher at a small table by a window.

Mrs. Elmore with Her Fourth Period Algebra I Class

Sally Elmore's shoulders sank. "Why is this so hard for them? They're eighth graders in the above-average track," she thought to herself. "And this is something we've talked about a zillion times. They sure knew how to do linear functions from the old textbook."

"Mrs. Elmore. Mrs. Elmore! *Over here!*" Tom was bellowing and waving his hand as if he were waving down a train from across the room. "Just a minute, Tom," Mrs. Elmore said with a bit of exasperation in her voice. "I am still helping Andy and Sam."

A few minutes before, Mrs. Elmore had placed a problem from one of the reform curricula on the overhead projector for her fourth period Algebra I class. Entitled "The Raven," the problem was an experiment intended to strengthen student understanding of linear functions.

Essentially, the investigation involved starting with a clear cylindrical vessel, pouring in an arbitrary amount of water, depositing marbles of uniform size into the container, measuring the height of the water level, and plotting the height as a function of the number of marbles in the container. Today Mrs. Elmore wanted her students to actually experiment and record their observations, hoping that the experiment would help them move from a graphical to a functional representation of the relationships between marbles and water height.

The activity took its name from an old Native American legend in which a raven, dying of thirst, drops pebbles into a deep well until the water level is raised to the point where it may be reached by the bird's beak. Mrs. Elmore had divided her class of 28 students into several cooperative learning groups. Though the students at the middle school often socialized when in these groups, she still felt it was important to group students this way. The groups each received ordinary glass jars of undetermined volumes.

Mrs. Elmore moved over to a group with Tom, Veda, and Becca. "So what's the problem?" she asked the group. She could see that on their papers, the trio had made a table with columns labeled "marbles" and "water" (see Figure 2.2).

Mrs. Elmore looked at their glum faces. "Okay, so now what? What do you think about when you see columns of points like that?"

"A line?" said Becca reluctantly, after a long silence.

"Yes, and what does a line always have?"

Tom looked confused. "Length?" he said with a tentative expression on his face.

"Yes, but what else?" Mrs. Elmore said. "What does a line always have?"

"Oh, I know!" piped up Veda, "*Slope!*"

Figure 2.2 Tom, Veda, and Becca's Work

Marbles	Water
0	$10\ ^1/_2$
2	11
4	$11\ ^1/_2$
6	12
8	$12\ ^1/_2$
10	13

Mrs. Elmore felt relieved that at least one of the students seemed to remember that they were in the midst of a unit on slope, intercept, and linear equations. "So," Mrs. Elmore continued, "if this line has a slope, what is it?"

Becca made an ugly face. "What's the matter, Becca?"

"We already talked about that, but we don't agree."

"What do you mean, you don't agree?" she inquired, peering at the threesome.

"We get different slopes and Tom says that's OK because we used different points. I don't think it's OK," Becca responded.

"If you want to get the same slope, you haf'ta use the same points," Tom affirmed. Becca sighed, "But Veda and I say it don't matter which points we use. Who's right?"

Mrs. Elmore, remembering how her workshop leader had acted, returned the question with a question: "Well, why don't you try again using different sets of points? Tom, why don't you use (0, 10½) and (10, 13), Becca you use (2, 11) and (6, 12), and Veda you try (4, 11½) and (8, 12½). See what you come up with."

After a couple of minutes' calculation, Becca and Veda both computed values of ¼. Tom, on the other hand, found a slope of .35.

"So," said Becca, looking at Tom's paper, "I guess Tom is right. You gotta use the same points to get the same slope."

Seeing the two answers of .25 and .35, all of a sudden Mrs. Elmore wasn't sure. *Did it make a difference?* she said to herself. Caught up with the question and feeling quite uncomfortable, she was jolted out of her confusion by Veda's giggle.

"Hey, Tom!! Where'd ya learn to subtract?" Veda said with big grin on her face.

"What do you mean?" Tom shot back, a bit defensive. "13 minus 10½ is 3½."

"Since when?" Veda continued. "13 minus 10 is 3. 13 minus 11 is 2. What's 13 minus 10½?"

"2½," interjected Becca before Tom could say anything.

Mrs. Elmore realized what had happened and said, "Maybe you forgot to borrow one from 13 and rewrite it as 12 and 2/2, Tom. Do you see that?"

"Yeah," he said with a deep sigh.

"So what's 2 and ½ divided by 10?" Mrs. Elmore continued.

Tom still looked puzzled. "Do I have to invert and multiply or is it one of those where you look for common . . . what are they called?"

"What do you think?" Mrs. Elmore responded.

"Which one do I flip—or can I just cross multiply?"

Now it was Becca who looked exasperated. "Geez," she said, "2 and ½ is the same as 2.5 and when you divide by 10, you just move the decimal point one place. So it's .25. Okay?"

Tom's looked down at his paper and quietly said "Oh yeah, I get it now."

"Good, ¼, the same as Becca's and Veda's calculations. Now do you think it matters which points you use to find the slope?" Mrs. Elmore continued.

"I guess not," answered Tom with a certain resignation. "But I still don't see why not," he added under his breath.

Ignoring Tom's comment, Mrs. Elmore pressed on, aware that the period was nearly over. "So now, what does slope mean in this investigation? Just what is slope?"

Dead silence. Nothing.

Becoming a bit impatient, Mrs. Elmore said, "Remember, slope is rise over . . .

"Run!" exclaimed Tom, now looking somewhat vindicated.

"Yeah," Becca exclaimed. "One marble makes the water go up four spaces."

"Yup, that's right," said Veda. "It's got to be right because it's rise over run, that's what you always say, huh, Mrs. Elmore? So since it makes the water rise four spaces, it's four over one . . . RISE OVER RUN."

Mrs. Elmore felt herself getting confused again. Veda certainly sounded convincing.

"But what's that .25 that we got?" Tom asked looking back at his paper.

"Let's see if we can make some sense of this." Mrs. Elmore tried to sound excited and confident about the search for understanding. "How much water did you start with? 10½, right?"

"Now when you added two marbles, how high was the water?"

"11 centimeters," Veda said.

"So if 2 marbles made the water rise ½ centimeter, could 1 marble make it rise 4 centimeters, Tom?"

Jolted back to the reality of the classroom, Tom looked first at the girls and then at Mrs. Elmore,

and said "No," though it was clear he didn't have a clue as to what the question was.

"You're sure?" Mrs. Elmore inquired. Tom said nothing.

"OK, how about you two?" Mrs. Elmore said, glancing at the girls. Both nodded their heads in assent.

"So it takes 4 marbles to make the water rise 1 centimeter. Explain to me what slope means," Mrs. Elmore said, returning to the original question.

"It means that the height of the column of water goes from 10½ to 14½," Veda quickly responded.

"What do you think, Becca?" Mrs. Elmore asked, smiling.

"That might be right, but I also think it means that four marbles make the water go up 1 centimeter."

"Is that the same as if someone said that one marble made the water level rise ¼ centimeter. Is that the same?" Sally asked.

"Yeah," said Becca, in a totally bored tone of voice.

"And you agree, Veda?"

"I think so . . . I guess . . . well, I'm not really sure. I think I'm getting confused about all of this. Maybe I'll come see you after basketball practice. Would that be Okay?" Veda asked.

"Not today, Veda. I have one of those math workshops right after school. How about tomorrow?"

"Okay," Veda replied as the bell rang.

Reflecting on the class, Mrs. Elmore realized that some students who wrote linear functions perfectly well using the algorithms in the old textbook were now floundering, and she wasn't sure why. Did the activity only confuse rather than help her students understand the concept? In her plan book, she juggled classes around to give the Raven problem one more period, in the hope that Tom and Becca would gain the same understanding and confidence that Veda had.

Tracking at West

Mrs. Elmore's Algebra I class was an above-average class, but she wasn't sure what she thought about tracking. Even though the math department still tracked their students, Mrs. Elmore knew that the plan was to move toward heterogeneous grouping over the following 2 years. The math department was the only department that still tracked students at West. Many members of the math department thought it was a mistake to move to heterogeneous grouping, citing the different social backgrounds and the range of abilities and knowledge of the students. One teacher, Abe Morgan, even went to a local state college library to see if there was any solid evidence about what was best for kids. Unfortunately, the only studies he could find were filled with statistical equations, *t*-tests, and education jargon. He declared the articles "Worthless and unintelligible!" He continued, "Even if you could even understand that stuff, I'm pretty sure there are just as many studies on both sides about tracking!"

After a few discussions in the teachers' room, Mrs. Elmore learned that it was best for her to keep her mouth shut about this issue since feelings ran so high, especially among the "veterans." Though her background as an elementary school teacher had led Mrs. Elmore to believe that valuable social skills were learned in heterogeneous groups, it seemed to her that the range of knowledge, skills, and ability in math was much greater in the middle school classes. Perhaps the range, she reasoned, was not due to different academic abilities, but rather to the fact that children came from different elementary schools where some teachers had not taught certain topics.

Workshop #2: Despair for Alison and Redux for Mrs. Elmore?

Alison Tansey happened to be at West in the afternoon and decided to pop into the second Making Sense in Middle School Math workshop to see how things were going. When Alison arrived at the math workshop, she briefly met the workshop leader and then took a seat in the back of the room. As a warm-up exercise, the leader asked the teachers to carry out a small activity about standards: "Which of the following statements reflects what you believe about standards? A, B, or C?" On the overhead projector, the leader placed the following:

A. The same set of high standards should be set for all students, and they must meet them in order to be promoted or to graduate.
B. Different standards should be set for different students, depending on the students' different interests and abilities, and they must meet these

differentiated standards in order to be promoted or graduate.

C. High standards should be set as a target, but with the recognition that all students may not necessarily achieve them. No one will be held back if they do not meet these standards.

"Once you've made your decision, go to the corner with the letter that best reflects your beliefs about the standards movement," he said.

Sally Elmore was sitting with Iris in the middle of the group. She could see that large red letters had been posted in three corners of the room. She knew what she believed. She believed in differentiation, even if she was not sure how to manage it in the classroom all the time. She was not the first to stride to her corner, but she did walk confidently to corner B. In a few moments, most of her colleagues had joined her. Nicole and two others stood in corner C.

"Okay, let's talk about this," the leader said. In the ensuing discussion, people disagreed vehemently with one another and the leader; two people left the room. Nicole Wittershein argued clearly and persuasively that students must master the basic skills before they could grapple with concepts such as variables.

Mrs. Elmore was upset at the way the leader set up the exercise. Almost everybody had gone to her corner, and the leader basically told them they were wrong. He said, "*Let's* talk about this," but he didn't let anybody get a word in edgewise. It made her angry that the leader asserted that *his* instructional strategies would help kids, when all he was doing was humiliating teachers and modeling strategies that certainly were not what students needed—not in her class, anyway! At the end of the discussion, the leader stated, "Tracking slower students into different ability math classes doesn't help them. They'll never be exposed to the math they need to be successful citizens in those dummy classes."

An Integral Part

It was not dedication to the ideas promulgated in the workshop, but commitment to her students that made Mrs. Elmore stick with the Raven problem about marbles and water. In her next meeting with her fourth period Algebra I class, she found herself moving toward Andie and Sam, who were busy talking about the dance being planned for Thanksgiving. As she approached, they feigned attention to the task at hand. Andie and Sam had begun their experiment with a different sized beaker from Tom, Becca, and Veda's group. It had about 4½ cm of water in it when they began.

Looking at their papers, Mrs. Elmore noticed that both Andie and Sam had the same table of values but their graphs didn't look the same: Sam had connected his points and Andie hadn't. Again remembering that researchers often said it was best to have students explore, she asked, "Do you think your graphs are the same?"

"Sure, Mrs. Elmore," Andie replied. "We used the same points."

"You don't always connect 'em when ya graph," Andie said, her brown eyes sparkling with confidence as she wrote with one hand and twirled her shoulder-length hair with the other.

"You don't, Andie? Can you tell me why?" Mrs. Elmore inquired.

"Nope. I just know ya don't." She unconsciously snapped her gum as if to add emphasis.

"Just think a minute," Mrs. Elmore continued. "Don't you remember that two points determine a line? You should connect them, Andie."

"Nah, I'm not sure, Mrs. Elmore," Andie said. "Look here on Sam's graph. What's the water level at this point?"

Sam squinted and drew his finger horizontally across the graph to the vertical axis. "I dunno. About eleven?" he answered questioningly.

"That's right, just about eleven centimeters," Andie replied. "Now, how many marbles are in the jar when the water is at this level?"

Mrs. Elmore put her finger on the same point as Sam had, but instead of sliding her finger across the paper, she drew it straight down to the horizontal axis. "About four and a half," she said with confidence.

"But that doesn't make any sense!" Andie replied. "Who ever heard of a half of a marble?"

At that moment, Sam and Andie could scarcely contain their merriment at seeing Mrs. Elmore so completely flummoxed. She vaguely recalled a problem that she had seen that asked about the number of buses needed to carry students to a football game. Was this the same kind of problem, Sally wondered?

Time and Time Again

After her interaction with Andie and Sam, Mrs. Elmore turned to look at the work of a third group of students—Liz, Greta, and David. She noticed they had yet a third set of points because their container was a different size from the other groups (see Figure 2.3).

She felt better about this group because they usually had the right answers and seemed to be strong in their computational skills. She could see that they, at least, were making the transfer to functional notation in their work. "What'd you get for your equation of the line?" Mrs. Elmore asked.

"We got something like $y = (.45/7)x + 4\ 2/5$ for the rational form and $y = .064x + 4.4$ for the decimal," David replied.

"Good. So what do x and y represent?" Mrs. Elmore asked.

Figure 2.3 Liz, Greta, and David's Work

Marbles	H_2O
0	4.4
7	4.85
14	5.3
21	5.75
28	6.2
35	6.65
42	7.1
49	7.5

"x is how many marbles you put in," answered Liz.

"And y, Greta?"

"How much the water goes up when you put the marbles in," Greta responded.

"How much did the water rise when you added seven marbles?" Mrs. Elmore pressed a little bit, remembering that it was a good technique to push students until you were certain they really understood.

"4.85 cm," Greta answered.

"Okay," She accepted the answer but continued, looking at Liz for confirmation, "Are you sure of Greta's answer?".

"No," replied Liz honestly, "but I think so. I don't see what else it can be."

"Well, Greta's right," Mrs. Elmore stated cheerfully and with relief that at least one of the groups in her class seemed to be getting the point of the exercise. "x is the number of marbles, and y is how much the water rises."

Just then the bell rang. Mrs. Elmore rushed to the board to write down the homework, but with half of the students already out of the room, she realized it was hopeless.

QUESTIONS

1. What do you think of Sally Elmore's decision to use "The Raven" problem?
2. Describe the content and approach of a professional development program that would help a teacher like Sally.
3. Should Mrs. Elmore be teaching at the middle school level? Why or why not?