CASE 11

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It's Time for a Tail

PRE-CASE EXERCISES

Please answer the following questions in preparation for the case discussion.

1. If you flip a coin once, what are the possible outcomes?
2. If you toss a coin twice, what are the possible outcomes?
3. If you toss a coin 6 times, which sequence (if any) is most likely to occur? Explain your reasoning.
   HHTHTT HHHTHH THTTHT TTTTTT
4. If you put tiles numbered 1 through 10 in a bag, reach in and randomly select one, what is the probability that the number you select will be greater than 7?
5. Does streak shooting have an influence on the probability of the next shot?

THE CASE

Ms. Brady looked at some of her students' answers on the worksheet she had developed (see Figure 11.1). Not including Tom, who argued that it didn't matter, two thirds of her students thought Michael should take the last shot while one third believed that Dennis would be a better choice. Their reasons, however, indicated a range of understandings. Even after 2 weeks on the topic, including coin tosses, lotteries, M&Ms in a dish, and marbles in a bowl, Ms. Brady was amazed. "What else can I do?" she asked herself.

Ms. Brady was disappointed that her worksheet hadn't produced a more lively discussion among her students, since streak shooting was a common event that actually was hotly debated in mathematics. She had hoped that the sheet would initiate a debate between those students who believed in streak shooting and those who took a more mathematical view. As she contemplated whether or not to move on to other topics under the relentless pressure of the curriculum, she reviewed the day in her mind.

Sixth Period Algebra I at Granville North

"Hey, lets get going!" Ms. Brady said lightheartedly to her sixth-period algebra I class. She waited a few moments for the students to finish their conversations, organize materials, and find their seats. "Take out your homework, and let's go over the assignment." About half the class began searching for their work. Matt and Jerome opened their books, choosing this moment to start the assignment, while the others in the room simply sat motionless or quietly continued their conversations.

Granville, once a largely White working- and middle-class city near a large metropolitan area, was home to an increasing number of Asian, Hispanic, and eastern European immigrants, most of whom arrived knowing little or no English. In addition, many of the blue-collar jobs in the city had disappeared, and Granville found itself changing to a commuter city. Granville's population supported two high schools—Granville High, which was strictly academic; and Granville North, which housed an academic program and a technical/vocational school. Jean Brady and the other members of the Granville North school community were well aware that they would always be considered inferior in the inevitable competition between the schools, a rivalry that spilled over into the town as a whole.

The staff at Granville North was veteran and most comfortable with the traditional teacher-cen-

Figure 11.1 Ms. Brady's Worksheet

Algebra I
11/99

Name:
Partner:

How Hot Is Your Shot?

Here are the facts:

Dennis makes 5 out of 10 baskets (an average).
Michael makes 7 out of 10 baskets (an average).

1. a. What is the probability that Dennis will make a basket?

   b. What is the probability that Michael will make a basket?

Imagine the following situation:

You are the coach of the basketball team, down by one point with the play remaining in the game. Your team has the ball. Dennis has made all of his last five shots, while Michael has missed his last three. Who should make the final shot? Why?

tered classroom requiring limited interactions between student and teacher or between students. As a second-year teacher, Ms. Brady tried to be much more innovative. She found this to be a challenge, as most of her students were unaccustomed to justifying their answers verbally or interacting with classmates. A question posed to her class could turn into a shouting match, as students would assert their "answers" with incredible force and often rude language. She sometimes wondered if she could overcome the dominant culture by herself.

The Repeats

Today appeared to be a full class, in what was a "catch-all" algebra I group of 20 students. The class was mostly made up of tenth-grade algebra "repeats"—students who had failed algebra the previous year—as well as students who were on the "slow track," coming from an intro to algebra or basic math course in the ninth grade. These students were intentionally placed in the class because they were older than the incoming freshman algebra students. The class also contained a few seniors who had avoided math until now and needed the course to fulfill the school's graduation requirement.

It was a boisterous group, often generating good questions about math, but also always looking for distractions and ways to avoid the work. Homework and organization were a major struggle for more than half the class. Ms. Brady required the use of a three-ring binder and provided assignment sheets listing the week's work to assist students. However, homework was often undone, incomplete, or lost. Today she decided to go over the assignment in class and then collect it afterwards, hoping to gain a few more participants in the discussion.

"Okay, please stop talking and look up here." She waited. Finally, she seemed to have everyone's attention. "Let's go over the homework together, and then I will collect it. Open your book to page 41, numbers 1–6. If you don't have your book, look on with someone else."

"Let's look at number 1. How do we find the probability of an event? Raise your hand if you know." A couple of hands went up while Ms. Brady wrote on the overhead:

the probability of an event =

"Only Tom and Christie know how? Who else knows?" A few more hands went up. "Okay, David, let's hear your explanation."

"Three fifths," David said clearly. David, usually alert in class, could be relied upon to solve some of the more difficult mathematical problems, but would become frustrated when attempting to explain how he arrived at his answer.

"You're giving me an answer, but how did you find your answer?" Ms. Brady urged.

David was silent, while a few other students began yelling out answers and talking out of turn. "Whoa! Hold on, let's look at the question again. If a number is chosen at random from the following set of numbers {1, 2, 3, 4, 5}, what is the probability that it is less than 4?" she read aloud. "What does 'chosen at random' mean?" Ms. Brady redirected and called on Christie, who had again raised her hand.

"It means that it can be any one."

"Any one of what?"

"Any one of the numbers—at random."

"Good. If something is chosen at random, then we have no idea what number is going to be chosen, but it is going to be 1, 2, 3, 4, or 5. Now I want you to think about finding the probability that any one of these numbers is less than 4. First let's remember how to find the probability of any event." Ms. Brady turned back and again wrote on the overhead projector (see Figure 11.2).

"How many possibilities are there in this case?" she asked the class.

A few students yelled "Five!" Everyone appeared to be in agreement.

Figure 11.2 The Overhead

$$\{1, 2, 3, 4, 5\}$$
$$x < 4$$
$$\text{the probability of an event} = \frac{\#\text{ winners}}{\#\text{ possibilities}}$$

"How many are winners?"

To this question she got a few different responses as students continued to yell out answers.

"Okay, Okay, quiet down. Raise your hands."

Ms. Brady wrote the answers she received on the overhead—a 4, a few 3s, and a 1 from Benjamin. She remembered her conversation with Benjamin a few days earlier about a coin-toss problem. The class had been finding the probability of tossing a coin twice and having the first toss be a head, the second toss be a tail as an outcome. Benjamin had difficulty understanding that there were four possible outcomes, and insisted that there were only two, head or tails. To his credit, Benjamin maintained his point of view despite the insistence of his friends to just give the correct answer. Now he seemed to be having a similar difficulty.

"So how many of the numbers from 1 through 5 are less than 4? One, two, and three are less than 4. Is 4 less than 4?" Ms. Brady addressed the class.

Quite a few hands went up. Jerome blurted out, "No, it's not, so the answer is 3." Then he put his hands on his hips and made a face. "Everyone knows that, huh, Ms. Brady?" Jerome enjoyed being the class clown, often making exaggerated gestures and loud comments. Ms. Brady added the answers on the overhead (see Figure 11.3).

Ms. Brady could see that Benjamin was still frowning. "What's the problem, Ben?" she asked.

"But there's only one winner," Benjamin said with a note of frustration in his voice.

Figure 11.3 More Information Is Added

$$\{1, 2, 3, 4, 5\}$$
$$x < 4$$
$$\text{the probability of an event} = \frac{\#\text{ winners}}{\#\text{ possibilities}} = \frac{3}{5}$$

"Yes, but how many possible winners are there?" repeated Ms. Brady.

"One," Benjamin maintained.

"Would three be a winner?"

"Yah."

"Two?"

He nodded.

"Well, you've named two possible winners. How many more are there?"

Benjamin thought a moment. "Uh, there are three less than five." He paused again and then said, a bit under his breath, "But I still think there is only one winner."

Ms. Brady noticed a few members of the class having their own discussions, so she returned to her overhead, discussed the next problem, and then collected their papers. About 30 minutes was left in the 65-minute period.

"Okay," said Ms. Brady, looking at the collection of awkward bodies stuffed into the one-armed chairs. "Find a partner to work with for the next 15 minutes. I have a worksheet for you to work on with your partner." Ms. Brady spoke to the entire room as she passed out one sheet of paper to each group. "Please write both names at the top."

What Does a Coin Know?

Approaching Katie, Donna, and Christie, Ms. Brady looked at the paper that they were working on (see Figure 11.4).

"Is this right?" Christie asked, chewing gum and looking up at Ms. Brady.

Looking at the work, Ms. Brady could see that they had found the correct probability for the coin tosses in the first problem, but that they were having trouble with the second question. She had deliberately designed the second question to be similar to what they had done in class, yet requiring a new application of what they knew about coin tosses.

Donna and Katie were extremely quiet, rarely saying anything in class. They looked silently at the paper.

"Okay, let's see what we have here. So, you think that it is more likely for the next toss to be a tail. Why?" She directed her question to the trio. Katie spoke up.

"Because it's time for a tail."

"Oh? Why is it time for a tail?" Ms. Brady redirected.

"So then it's a head," Christie said, looking intently at Ms. Brady's face.

"Explain why a head is more likely," Ms. Brady urged.

"Well, it's more likely to be a head because it's been heads every other time," Christie spoke again. Ms. Brady noticed that Donna was busy crossing off one answer and writing another.

"Hold on. A coin is being tossed. What is the probability that it will be a head?" Ms. Brady asked.

"50–50," Christie said with certainty.

"Yes, now you toss that same coin again. What is the probability that it will be a head?"

With a slight hesitation, Christie said, "50–50."

"Does the coin remember that it has been tossed before and came up heads?"

Thoughtful looks and sly smiles appeared on all the faces. They came to the consensus that the coin wouldn't remember.

"So, even though I've tossed a coin five times, and each time it comes up heads, what is more likely on the next toss, a head or tail?" Ms. Brady inquired again.

"Either one," Donna said quietly, still with a hint of disbelief in her voice.

Just then Ms. Brady noticed that several students were off-task and that any productive engagement with the worksheet was over. The students had only been working for 10–15 minutes. She encountered this problem frequently with this class. They would work well for a portion of the lesson and then they would fall apart. Before things went any further, Ms. Brady stopped the activity, telling the class to return to their assigned seats.

Assessing the climate of the room, she made a quick change of plans and decided to use the worksheet "How Hot is Your Shot?" (refer to Figure 11.1) as an individual assessment of the lesson instead of continuing with another partner activity. Using a firm voice, Ms. Brady instructed the students to complete the worksheet in silence. Students quieted down quickly and wrote their responses.

The End of the Tail?

Still seated on her comfortable couch, Ms. Brady now looked at the "How Hot is Your Shot?" papers, taking some time to read the responses

Figure 11.4 Katie, Donna, and Christie's Work

Algebra I
11/4/96

Name: DONNA
Partner: KATE
CHRISTIE

Find the probability of the following events and explain your reasoning.

1. coin toss:

   a. one toss: a tail 1/2

   b. two tosses: a head, then a tail 1/4

   HH TH
   TT HT

2. You've tossed a coin 5 times and had the following outcome:

   HHHHH

   a. Which is more likely on the next toss, a head or a tail?

   ~~tail~~ ~~head~~ either one 50 50 chance

   b. Why? Explain your reasoning.

   ~~because its been heads every other time - so its more likely to be head~~

   because the coin does not Know how many times we have tossed it

3. lottery drawing from the six numbers: 1,2,3,4,5,6:

   a. your number 453261 (pick a number)

   b. which is more likely to be drawn, 123456 or your number

   either one

   c. Why? Explain your reasoning.

   because it has a 50/50 chance

   (720)

4. If there is a 40% probability of snow today, what is the probability that it will not snow today?

   60% chance of not having snow

carefully. All of the students answered the first part correctly, 50% and 70% respectively. But their responses to the streak-shooting situation varied:

Ashley: *Michael should make the last shot because he has an average of 7 out of 10. He already missed 3 so the probability that he'll miss that shot isn't very bad. Dennis has already gotten 5 in and he has and average of 5–10 so he will probably miss his next shot.*

Benjamin: *Dennis should, because his percentage is better for this game in particular. His chances are better than Michael's.*

Donna: *Dennis should because he made all of his last 5 shots.*

Katie: *Dennis, because he's been making all of his other shots.*

Steve: *Michael, because he is due for a basket, because he missed all the others.*

Jerome: *Mike, because he will make the next seven.*

Tom: *I don't think it matters who shoots it, because the chances of getting the ball in is one out of two, in other words a 50% chance either of them would get it in.*

Christie: *Michael should make the basket since he has missed 3 than 7 more should make it.*

Ms. Brady sighed as she looked for more insight into her students' understanding of probability, and wondered to herself about streak shooting. Who would be more likely to make the next basket?

QUESTIONS

1. Should Ms. Brady continue with probability in this particular class or move on to other topics in the algebra curriculum? Why?
2. What specifically would you say to Benjamin? To Steve?
3. Do you believe in "streak shooting"? Defend your position.
4. What can you do to maintain positive momentum in a math class?