ALGEBRA for All

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Algebra for Students with Learning Disabilities

Bergen Community College (BCC), like many schools and colleges, is facing a growing challenge of providing appropriate instruction to students with learning disabilities. Such accommodations as tutoring, untimed testing, note-takers, and readers have been used with some success. However, these accommodations are insufficient to ensure success in mathematics courses for most students with learning disabilities.

A disproportionate number of students with learning disabilities enrolled at BCC have graduated from high school with little exposure to algebraic concepts. When I ask my students with learning disabilities why they did not enroll in an algebra class in high school, their responses are almost always the same.

- My counselors assumed that I would not pursue a college education, so why study algebra?
- Since I had difficulties in my previous study of mathematics, my counselor thought that I did not need the frustrations and the possible failure associated with learning algebra.
- My counselor told me that studying general mathematics or prealgebra, in which I could review the basic mathematical concepts of my elementary school years and apply them to life skills, would be more beneficial.

Counselors apparently believe that learning-disabled students do not need or cannot learn algebra

Counselors apparently believe that students with learning disabilities do not need or cannot learn algebra. Therefore, algebraic concepts are not made accessible to them. But with the need for postsecondary education in today's society and the open-enrollment policy of community colleges, many students with learning disabilities are continuing their education beyond high school.

Because of their limited exposure to college-preparatory mathematics, students with learning disabilities enter BCC underprepared in mathematics. Yet if they are enrolled in a degree program, they must successfully complete a basic algebra requirement, at a minimum. With the population of such students increasing each year, restructuring the classroom environment to meet the needs and learning styles of these students has become necessary. Since fall 1996, BCC has offered designated sections of algebra for students with a documented learning disability. At BCC, a learning disability is documented by (1) high school records that include a psychological report, a social history, an educational report, and an individualized educational plan; (2) a neuropsychiatric or neuropsychological report; or (3) a state-supported testing report. The requirements and content of the learning-disabled (LD) algebra classes are the same as those of the regular basic algebra course. These sections differ, however, in the structure of the class and the teaching strategies and modes of instruction used in the classroom.

Our success rates indicate that students with learning disabilities can learn algebra when the environment focuses on their needs and learning styles. Passing rates in the LD algebra classes have ranged from 60 percent to 80 percent, rates that are comparable with, if not better than, passing rates

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for all basic algebra classes at BCC. The problems encountered by a student with learning disabilities in studying algebra are the same as the problems encountered by the average student; however, they are more pronounced. In fact, most students in the LD algebra classes have more difficulty with numerical computation than with algebraic concepts.

THE LD ALGEBRA CLASS AT BERGEN COMMUNITY COLLEGE

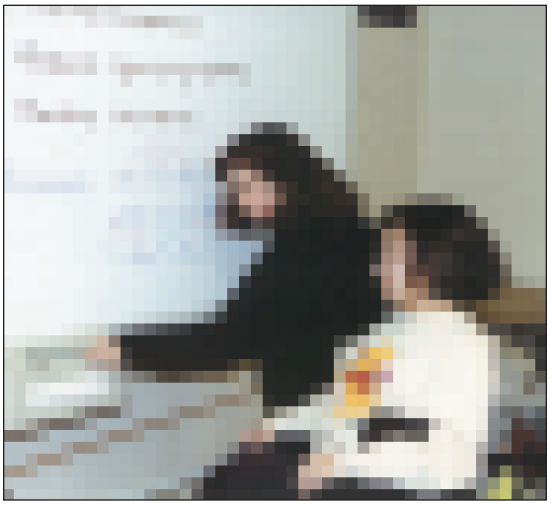
The elementary algebra program at Bergen Community College is a fairly traditional high school first-year-algebra course. The program consists of two tracks; one is a four-credit one-semester course for students who need to review their basic algebra skills, and the other consists of two three-credit sequential courses, equivalent in content and requirements to the four-credit course, for students with more pronounced weaknesses in algebra. The elementary algebra program is mastery based. Students must successfully complete 80 percent of the course objectives, with one retest permitted on each objective. Grades are determined by points earned on objective tests. However, to receive the earned grade rather than a failing grade, students must pass an exit examination when they complete the elementary algebra program.

Students with a documented learning disability who need to improve their algebra skills are encouraged to enroll in the two-semester LD algebra sequence. The academic counselor for students with disabilities places students in these sections.

THE STRUCTURE OF THE LD ALGEBRA CLASS

The structure of the class and the classroom environment have been major factors in the success of the LD algebra courses.

- Since many students with learning disabilities have difficulties focusing for extended periods of time, class meetings for these special sections of algebra are limited to one hour. I vary classroom activities from instruction and discussion to individual and small-group problem solving to accommodate students with attention-deficit disorder or hyperactivity. I frequently have students do problem-solving activities at the chalkboard, where those with graphomotor problems, that is, poor handwriting or difficulty in copying written work, or those with organizational problems have sufficient space to write their work in a readable, orderly fashion.
- Enrollment in these designated sections is limited to twenty students, enabling me to work one-on-one with students in developing strategies and gimmicks that tap their strengths. At times, a special trained peer tutor is present in the classroom to guide and support the weaker students.



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- The accommodation that is most frequently furnished to students with learning disabilities at BCC is untimed testing, which takes place outside the classroom, usually in the testing center. Since the instructor is not available during such testing, the student loses the opportunity to ask for clarification of test questions. To rectify this situation, I schedule one additional class meeting each week to allow time for in-class testing. This format eliminates the need for readers for students with dyslexia, because I read to students who have decoding or comprehension problems when they need such assistance. Incorporating the extra hour of class time into the schedule also gives the time needed to develop individual learning strategies.
- To accommodate students who have difficulties taking notes, I give all students lecture notes that include definitions and explanations, as well as examples. An electronic blackboard, a blackboard with a built-in printer that copies anything written on its surface at the press of a button, is available during classroom instruction to enable me to give students a hard copy of information written on it. This hard copy appears to be important to their understanding of the concepts and their recall of information.
- The prevailing philosophy for educating students with learning disabilities is mainstreaming whenever possible. However, students enrolled in these designated sections of algebra find the homogeneous class to be a comfortable, non-threatening, and supportive environment. A spirit of cooperation develops. Students no longer feel like the "dummies" of the class; all students are "in the same boat." As a result, students are free to ask and answer questions, as well as to help their classmates.

Students find this homogeneous class to be comfortable and non-threatening

- As the classroom instructor, I believe that students with learning disabilities are capable of learning algebra when the teaching and learning strategies are appropriate. Support and encouragement enable my students to believe that they can be successful in an algebra class.

The classroom modifications used in my LD algebra classes can increase the success of students with learning disabilities in studying mathematics in any secondary or postsecondary educational setting. Educators might want to consider special sections of mathematics courses for students with learning disabilities. In these classes, the necessary accommodations are part of the classroom structure, as an alternative to mainstreaming at both the high school and the community college level.

I treat algebra as a foreign language with its own alphabet and grammar

TEACHING AND LEARNING STRATEGIES USED IN THE LD ALGEBRA CLASS

The teaching and learning strategies developed and used throughout the LD algebra class encourage the success of students with learning disabilities. Successful individual and group strategies include the following:

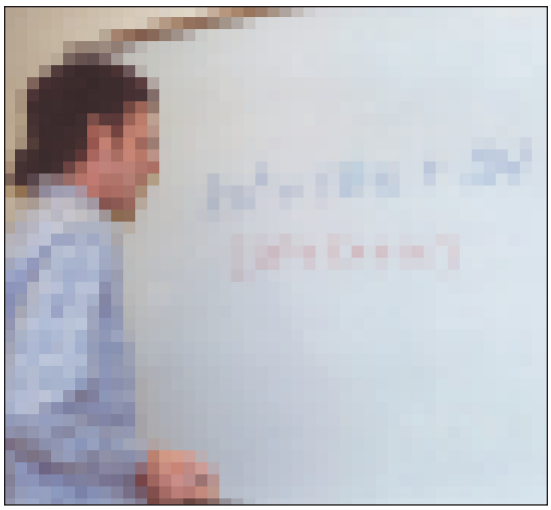
1. From the beginning of the course, I treat algebra as a foreign language with its own alphabet and grammar. I emphasize the proper reading and writing of this new language throughout the course.

   - Early in the course I emphasize repeatedly the distinction between terms and factors, for example,

     $\underline{2x} + \underline{5(x-4)}$ has two terms;

     $(x-1)(x+4)$ is one term consisting of two factors.

     Thus, when one of my students has to evaluate $4 - 2(-3)$, the student sees two terms in the expression and writes $4 + 6$ rather than $4(+6)$.



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- At the beginning of the course, I discuss the multiple use of parentheses, to indicate groupings or to indicate groupings and multiplication, for example, $6 - (8 + 3)$ versus $6(8 + 3)$. In $6 - (8 + 3)$, the parentheses indicate a grouping that says, "do me first and then subtract"; whereas in $6(8 + 3)$, the parentheses indicate a grouping and a multiplication and say, "do me first and then multiply."
- I am careful to point out the distinction between -3^2 and $(-3)^2$. Because we read from left to right, students often see the -3^2 as $(-3)^2$. To emphasize that 3 is the base in the expression -3^2, I have students insert parentheses and rewrite -3^2 as $-(3^2)$.
- I use what my students have dubbed the "imaginary 1" as a reading aid in simplifying algebraic expressions, for example,

$$2x + x = 2x + \mathbf{1}x,$$
$$a^3 \bullet a = a^3 \bullet a^1.$$

- Together, my students and I practice properly reading English phrases, discussing where to pause, so that they can achieve an accurate translation of both the symbols and the meaning of the language of mathematics, for example,

  the sum of a number and six less than the number is read *the sum of a number* PAUSE *and* PAUSE *six less than* PAUSE *the number* rather than *the sum of a number and six* PAUSE *less than the number*.

  Applications in which both verbal and nonverbal skills are important can be extremely difficult for students with learning disabilities. Translating the English language to the language of mathematics helps give a structured approach to solving numerical word problems. This structure enables my students to be successful in problem solving. This success carries over in a more positive attitude toward the applications covered later in the course.
- I require that all written mathematics indicate a one-step-at-a-time process, for example,

$$7 - 2(5 - 3) = 7 - 2(2) = 7 - 4 = 3,$$
$$2x(3x)(-4x) = 6x^2(-4x) = -24x^3.$$

- I insist on the proper terminology by all students at all times, for example,

$$\begin{array}{r} 2x + \cancel{1} = 7 \\ \underline{\quad -1 \quad -1} \end{array}$$

+1 and –1 *add out* to 0;

$$\frac{\cancel{2}x}{\cancel{2}} = \frac{6}{2}$$

2/2 *reduces* to 1.

All too frequently, students "cancel" +1 and –1 as well as 2/2. The words "add out" and "reduce" distinguish between a result of 0 and a result of 1, respectively.

2. Individualized learning strategies are developed as the need arises. Creativity is a necessary requirement in developing such strategies. Before working with the student on strategies, I must understand the nature of a student's error. If the error is mathematical, can the student and I devise an explanation that will clarify the misunderstanding? If the error is caused by the learning disability, can the student and I develop compensating techniques? The student must be involved in developing these learning strategies at all times. My participation, and perhaps that of other students, is secondary if the student is to take ownership of the strategy. Some individual learning strategies that my students have developed and used include the following:
   - Colored pencils are used to highlight relevant information. Negative signs are written in red for students who tend to have sign errors, terms in an expression are underlined in blue for students who have difficulties with the order of operations, and green arrows are drawn showing the distributive property for students prone to distribute incompletely.
   - I frequently remind my students to distinguish between the operations of addition and multiplication. Like things are added or combined; we add like numbers, for example, 3 tenths + 4 tenths; we add such like quantities as 7 feet + 2 feet; we combine like terms, for example, $5x + 8x$; and we combine like radicals, for example, $\sqrt{6} + 9\sqrt{6}$. However, both like and unlike things can be multiplied; we multiply 6 feet • 4 feet and \$3/foot • 2 feet, as well as $(2x)(3x)$ and $(5x)(2y)$.
   - Perseveration, or difficulty in stopping or modifying an activity, often interferes with the changeover in operations necessary when working with numbers and exponents. I recommend that students who perseverate pause before performing the second operation, for example,

     $2x^3 + 3x^3 = 5$ PAUSE x^3, so that the student does not write $5x^6$;

     $(2x^3)(3x^3) = 6$ PAUSE x^6, so that the student does not write $6x^9$.
   - We use rules minimally throughout the course; I instead emphasize understanding the concepts involved, for example,

     $-2 + 5 = 3$, since losing \$2 and winning \$5 results in a net gain of \$3;

     $x^2x^3 = x \bullet x \bullet x \bullet x \bullet x = x^5$.

     My motto in this particular class is, If in doubt, write it out. For example,

     $(5x^4)^2 = (5x^4)(5x^4) = 25x^8$;

     $\sqrt{x^7} = \sqrt{x^2 \bullet x^2 \bullet x^2 \bullet x} = x \bullet x \bullet x\sqrt{x} = x^3\sqrt{x}$.
   - I encourage students who tend to be aural learners to verbalize the steps of a process either to themselves or to other students.
   - For students with such reading difficulties as dyslexia or graphomotor difficulties, textbook problems using the letters b or t can lead to misunderstandings that are not mathematical. In such cases, I encourage students to use different letters for variables. For example,

     $(a^2b)(ab^3)$ is rewritten as $(a^2c)(ac^3)$;

     $3t + 5 = 11$ is rewritten as $3x + 5 = 11$.
   - When students evaluate algebraic expressions for given values of the unknowns, I encourage them to create a template; thus, when they substitute the values for the unknowns, they write the expression in correct algebraic notation. For example, to evaluate $2x - y^2$ for $x = -1$ and $y = -2$,

     (1) use the template $2(\quad) - (\quad)^2$,

     (2) replace x with –1 and y with –2,

     (3) evaluate $2(-1) - (-2)^2$.
3. Study skills and continuous review are integral parts of the course. Students use index cards to record basic processes, necessary formulas, or needed reminders. I always encourage them to use a "data dump," that is, to list reminders and important formulas on the test sheet from memory before beginning a text.

OUTCOMES

How have the "graduates" of the LD algebra class fared in subsequent mathematics courses? For many students enrolled in my LD algebra class, successful completion of this course fulfills their mathematics requirement. These students are enrolled in degree programs that require a science or mathematics elective, and the students usually choose to take a science course, such as biology. For others, technical mathematics or a general-education mathematics elective is required for their degree. With the support of peer tutors or mentoring by me, those who have taken my LD algebra class who have enrolled in these mathematics courses have successfully completed their mathematics requirements. The LD algebra class was the gateway to their degree for both groups of students.

Two of my students have enrolled in intermediate algebra, but they have not fared as well. Our four-credit one-semester intermediate-algebra course covers too much material in too short a time in too

The students should always be involved in developing their learning strategies

large a class for the student with learning disabilities. Because the number of students with learning disabilities who are required to take intermediate algebra is so small, we have not been able to establish a special section of this course for these students.

CONCLUSION

Motivation is the key to success

I have seen students with little exposure to algebra enter the LD algebra class scared, knowing that passing the algebra course is crucial to receiving an associate's degree. With hard work in an environment that is conducive to their learning, these students with learning disabilities have demonstrated that they can be successful in the study of mathematics by developing and using strategies that focus on their strengths. More important, they feel better about themselves both as students and as individuals.

The learning environment created in the special algebra classes, combined with the teaching strategies developed, has produced positive results for students with learning disabilities. A similar environment, with teaching and learning strategies that are based on student needs, can lead to a successful experience in any mathematics class, at all levels, for motivated students with learning disabilities. Yes, motivation is the key to success for any student. However, for a student with learning disabilities who has had few positive experiences in the mathematics classroom, early success in algebra, in conjunction with encouragement, can bring about the motivation needed for continued success. Moreover, the strategies developed by the individual student will be invaluable in studying other subjects and preparing for further education.

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